ECE 5273
Test 2

Friday, May 11, 2012
10:30 AM - 12:30 PM

Spring 2012

Dr. Havlicek

Name: SOLUTION

Student Num: ____________

Directions: This is an open notes test. You may use a clean copy of the course notes as published on the course web site. Other materials are not allowed. You have 120 minutes to complete the test. All work must be your own.

SHOW ALL OF YOUR WORK for maximum partial credit!

GOOD LUCK!

SCORE:

1. (20) ________
2. (20) ________
3. (20) ________
4. (20) ________
5. (20) ________

TOTAL (100):

____

On my honor, I affirm that I have neither given nor received inappropriate aid in the completion of this test.

Name: ____________ Date: ____________

1. **20 pts**. True or False. Mark *True* only if the statement is **always** true.

TRUE	FALSE	
	✓	(a) **2 pts**. Streaking and blotching often occur when the median filter is applied with a window that is too small.
	✓	(b) **2 pts**. Nonlinear image filters are almost always implemented by pointwise multiplication of DFT's.
	✓	(c) **2 pts**. The median filter removes positive impulses while ~~preserving negative impulses.~~
	✓	(d) **2 pts**. In image regularization, the smoothness constraint forces the smoothed version $(\mathbb{G} * \widehat{\mathbb{I}})$ of the restored image $\widehat{\mathbb{I}}$ to look like the sensed image $\mathbb{J}$.
✓		(e) **2 pts**. An advantage of the LoG edge detector is that edge thinning and edge linking are not required.
	✓	(f) **2 pts**. The Sobel edge detector is the best choice for detecting edges in noisy images.
✓		(g) **2 pts**. The main idea of anisotropic diffusion is to inhibit smoothing across edge boundaries.
✓		(h) **2 pts**. Additive white noise is characterized by a flat power spectrum.
	✓	(i) **2 pts**. The inverse filter is optimal for performing deconvolution when there is also additive white noise.
	✓	(j) **2 pts**. The "Wiener filter" is so named because it's inventor used to drive an Oscar Mayer Wienermobile.

2. **20 pts**. Consider the 2×2 digital images $\mathbb{I}_1$ and $\mathbb{I}_2$ shown below.

$$\mathbb{I}_1 = \begin{array}{|c|c|} \hline \mathbf{0} & \mathbf{1} \\ \hline \mathbf{4} & \mathbf{2} \\ \hline \end{array} \qquad \mathbb{I}_2 = \begin{array}{|c|c|} \hline \mathbf{7} & \mathbf{3} \\ \hline \mathbf{1} & \mathbf{5} \\ \hline \end{array}$$

Compute the circular (wraparound) convolution $\mathbb{J} = \mathbb{I}_1 \circledast \mathbb{I}_2$. Additional workspace is provided on the next page. Give your answer $\mathbb{J}$ in the space provided below:

$$\mathbb{J} = \begin{array}{|c|c|} \hline 17 & 29 \\ \hline 39 & 27 \\ \hline \end{array}$$

The work is on the next page.

More Workspace for Problem 2...

I_1 $i \downarrow$ "flip" of I_1

$$\begin{array}{cccc|c} \multicolumn{4}{c}{} & \\ 0 & 1 & 0 & 1 & -2 \\ 4 & 2 & 4 & 2 & -1 \\ 0 & 1 & 0 & 1 & 0 \\ 4 & 2 & 4 & 2 & 1 \\ \hline -2 & -1 & 0 & 1 & \end{array} \qquad \begin{bmatrix} 2 & 4 & 2 & 4 \\ 1 & 0 & 1 & 0 \\ 2 & 4 & 2 & 4 \\ 1 & 0 & 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix}$$

$j \rightarrow$ "flip" of I_2

$$I_2 \quad \begin{array}{c|cc} & 0 & 1 \\ \hline 0 & 7 & 3 \\ 1 & 1 & 5 \end{array} \qquad \begin{array}{cc|c} 5 & 1 & 1 \\ 3 & 7 & 0 \\ \hline 1 & 0 & \end{array} \qquad \begin{bmatrix} 17 & 29 \\ 39 & 27 \end{bmatrix} \quad J$$

$$J(i,j) = \sum_{m=0}^{1} \sum_{n=0}^{1} I_1(i-m, j-n) I_2(m,n) = I_1(i,j) I_2(0,0) + I_1(i,j-1) I_2(0,1) + I_1(i-1,j) I_2(1,0) + I_1(i-1,j-1) I_2(1,1)$$

$$= 7 I_1(i,j) + 3 I_1(i,j-1) + I_1(i-1,j) + 5 I_1(i-1,j-1)$$

i	j	
0	0	$J(0,0) = 7I_1(0,0) + 3I_1(0,-1) + I_1(-1,0) + 5I_1(-1,-1)$ $= 7\cdot 0 + 3\cdot 1 + 4 + 5\cdot 2 = 3 + 4 + 10 = 17$
0	1	$J(0,1) = 7I_1(0,1) + 3I_1(0,0) + I_1(-1,1) + 5I_1(-1,0)$ $= 7\cdot 1 + 3\cdot 0 + 2 + 5\cdot 4 = 7 + 2 + 20 = 29$
1	0	$J(1,0) = 7\cdot I_1(1,0) + 3\cdot I_1(1,-1) + I_1(0,0) + 5\cdot I_1(0,-1)$ $= 7\cdot 4 + 3\cdot 2 + 0 + 5 = 28 + 6 + 5 = 39$
1	1	$J(1,1) = 7\cdot I_1(1,1) + 3\cdot I_1(1,0) + I_1(0,1) + 5\cdot I_1(0,0)$ $= 7\cdot 2 + 3\cdot 4 + 1 + 5\cdot 0 = 14 + 12 + 1 = 27$

3. **20 pts.** Pixels in the 6×6 image $\mathbb{I}$ shown below take values in the range $\{0, 1, 2, \ldots, 99\}$. The image is sent through a communication channel where it is corrupted by additive noise. The received image $\mathbb{J}$ is also shown below.

$\mathbb{I} =$

11	**11**	**11**	**70**	**70**	**70**
11	**11**	**11**	**70**	**70**	**70**
11	**11**	**11**	**70**	**70**	**70**
11	**11**	**11**	**70**	**70**	**70**
11	**11**	**11**	**70**	**70**	**70**
11	**11**	**11**	**70**	**70**	**70**

$\mathbb{J} =$

11	**11**	**11**	**40**	**70**	**70**
11	**0**	**11**	**70**	**70**	**70**
11	**11**	**11**	**70**	**60**	**70**
45	**11**	**11**	**70**	**70**	**70**
11	**11**	**52**	**70**	**82**	**70**
11	**11**	**11**	**70**	**70**	**70**

(a) **10 pts.** Apply a median filter to compute the restored image $\mathbb{K}_{MF} = \text{MED}[\mathbb{J}, \mathbb{B}]$ and compute the ISNR. Handle edge effects by replication and use the window $\mathbb{B} = \text{SQUARE}(9)$.

(b) **10 pts.** Apply an inner average filter with $P = 2$ to compute the restored image $\mathbb{K}_{IA} = \text{INNER\_AVE}_2[\mathbb{J}, \mathbb{B}]$ and compute the ISNR. Handle edge effects by replication and use the window $\mathbb{B} = \text{SQUARE}(9)$.

Workspace is given on the next TWO pages.

Show the restored images here:

$\mathbb{K}_{MF} =$

11	11	11	40	70	70
11	11	11	60	70	70
11	11	11	70	70	70
11	11	11	70	70	70
11	11	11	70	70	70
11	11	11	70	70	70

$\mathbb{K}_{IA} =$

11	11	16.8	46.2	70	70
11	11	16.8	50.2	70	70
11	11	22.8	56.2	70	70
11	11	31	64.4	70	70
11	11	31	66.4	70	70
11	11	31	66.4	70	70

$\text{ISNR}_{MF} =$ 6.12996 dB

$\text{ISNR}_{IA} =$ 1.95903 dB

Workspace for Problem 3. . .

11	11	11	11	40	70	70	70
11	**11**	**11**	**11**	**40**	**70**	**70**	70
11	**11**	**0**	**11**	**70**	**70**	**70**	70
11	**11**	**11**	**11**	**70**	**60**	**70**	70
45	**45**	**11**	**11**	**70**	**70**	**70**	70
11	**11**	**11**	**52**	**70**	**82**	**70**	70
11	**11**	**11**	**11**	**70**	**70**	**70**	70
11	11	11	11	70	70	70	70

$\mathbb{K}_{MF}$

11	11	11	40	70	70
11	11	11	60	70	70
11	11	11	70	70	70
11	11	11	70	70	70
11	11	11	70	70	70
11	11	11	70	70	70

11	**11**	**11**	**70**	**70**	**70**
11	**11**	**11**	**70**	**70**	**70**
11	**11**	**11**	**70**	**70**	**70**
11	**11**	**11**	**70**	**70**	**70**
11	**11**	**11**	**70**	**70**	**70**
11	**11**	**11**	**70**	**70**	**70**

$K_{MF}(i,j) - I(i,j)$

0	0	0	30	0	0
0	0	0	10	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

$$MSE(\mathbb{K}_{MF}) = \frac{1}{36}(900+100) = \frac{1000}{36} = 27.7778$$

$J(i,j) - I(i,j)$

0	0	0	-30	0	0
0	-11	0	0	0	0
0	0	0	0	-10	0
34	0	0	0	0	0
0	0	41	0	12	0
0	0	0	0	0	0

$$MSE(\mathbb{J}) = \frac{1}{36}\left[(-30)^2 + (-11)^2 + (-10)^2 + (34)^2 + (41)^2 + (12)^2\right]$$

$$= \frac{1}{36}\left[900 + 121 + 100 + 1156 + 1681 + 144\right]$$

$$= \frac{4102}{36} = 113.944$$

$$ISNR_{MF} = 10\log_{10}\frac{MSE(\mathbb{J})}{MSE(\mathbb{K}_{MF})}$$

$$= 10\log_{10}\frac{4102/36}{1000/36}$$

$$= 10\log_{10}\frac{4102}{1000} = 6.12996\text{ dB}$$

More Workspace for Problem 3...

11	11	11	11	40	70	70	70
11	11	11	11	40	70	70	70
11	11	0	11	70	70	70	70
11	11	11	11	70	60	70	70
45	45	11	11	70	70	70	70
11	11	11	52	70	82	70	70
11	11	11	11	70	70	70	70
11	11	11	11	70	70	70	70

K_{IA}

11	11	16.8	46.2	70	70
11	11	16.8	50.2	70	70
11	11	22.8	56.2	70	70
11	11	31	64.4	70	70
11	11	31	66.4	70	70
11	11	31	66.4	70	70

11	11	11	70	70	70
11	11	11	70	70	70
11	11	11	70	70	70
11	11	11	70	70	70
11	11	11	70	70	70
11	11	11	70	70	70

$K_{IA}(i,j) - I(i,j)$

0	0	5.8	-23.8	0	0
0	0	5.8	-19.8	0	0
0	0	11.8	-13.8	0	0
0	0	20	-5.6	0	0
0	0	20	-3.6	0	0
0	0	20	-3.6	0	0

$2P+1=5$, so we average the middle 5 values in the window

$$K_{IA}(0,3) = \frac{1}{5}[\cancel{11}+\cancel{11}+11+40+40+70+70+\cancel{70}+\cancel{70}] = \frac{1}{5}\cdot 231 = 46.2$$

$$K_{IA}(1,3) = \frac{1}{5}[\cancel{11}+\cancel{11}+11+40+60+70+70+\cancel{70}+\cancel{70}] = \frac{1}{5}\cdot 251 = 50.2$$

$$K_{IA}(2,3) = \frac{1}{5}[\cancel{11}+\cancel{11}+11+60+70+70+70+\cancel{70}+\cancel{70}] = \frac{1}{5}\cdot 281 = 56.2$$

$$K_{IA}(3,3) = \frac{1}{5}[\cancel{11}+\cancel{11}+52+60+70+70+70+\cancel{70}+\cancel{82}] = \frac{1}{5}\cdot 322 = 64.4$$

$$K_{IA}(4,3) = \frac{1}{5}[\cancel{11}+\cancel{11}+52+70+70+70+70+\cancel{70}+\cancel{82}] = \frac{1}{5}\cdot 332 = 66.4$$

$$K_{IA}(5,3) = \frac{1}{5}[\cancel{11}+\cancel{11}+52+70+70+70+70+\cancel{70}+\cancel{82}] = \frac{1}{5}\cdot 332 = 66.4$$

$$K_{IA}(0,2) = \frac{1}{5}[\cancel{0}+\cancel{11}+11+11+11+11+40+\cancel{40}+\cancel{70}] = \frac{1}{5}\cdot 84 = 16.8$$

$$K_{IA}(1,2) = \frac{1}{5}[\cancel{0}+\cancel{11}+11+11+11+11+40+\cancel{70}+\cancel{70}] = \frac{1}{5}\cdot 84 = 16.8$$

$$K_{IA}(2,2) = \frac{1}{5}[\cancel{0}+\cancel{11}+11+11+11+11+70+\cancel{70}+\cancel{70}] = \frac{1}{5}\cdot 114 = 22.8$$

$$K_{IA}(3,2) = \frac{1}{5}[\cancel{11}+\cancel{11}+11+11+11+52+70+\cancel{70}+\cancel{70}] = \frac{1}{5}\cdot 155 = 31$$

$$K_{IA}(4,2) = \frac{1}{5}[\cancel{11}+\cancel{11}+11+11+11+52+70+\cancel{70}+\cancel{70}] = \frac{1}{5}\cdot 155 = 31$$

$$K_{IA}(5,2) = \frac{1}{5}[11+11+11+11+11+52+70+70+70] = \frac{1}{5}\cdot 155 = 31$$

$\longrightarrow$ NEXT PAGE

3(b) cont ...

$$MSE(\mathbb{K}_{IA}) = \frac{1}{36}\Big[2(5.8)^2 + (11.8)^2 + 3(20)^2 + (-23.8)^2 + (-19.8)^2 + (-13.8)^2 + (-5.6)^2 + 2(-3.6)^2\Big]$$

$$= \frac{1}{36}\Big[2(33.64) + 139.24 + 3(400) + 566.44 + 392.04 + 190.44 + 31.36 + 2(12.96)\Big]$$

$$= \frac{1}{36}\big[2612.72\big] = 72.5756$$

$$ISNR_{IA} = 10\log_{10}\frac{MSE(\mathbb{J})}{MSE(\mathbb{K}_{IA})}$$

Same as in part (a)

$$= 10\log_{10}\frac{4102/36}{2612.72/36}$$

$$= 10\log_{10}\frac{4102}{2612.72} = 1.95903\text{ dB}$$

4. **20 pts.** Consider the 1D "image" (signal) $\mathbb{I} = [1\ 2\ 4\ 1\ 3]$. Thus, $I(0) = 1$, $I(1) = 2$, $\ldots$ $I(4) = 3$. This image has **three**-bit pixels.

(a) **6 pts.** Apply a three-point gray scale morphological DILATION to $\mathbb{I}$ to obtain the result $\mathbb{J} = \text{DILATE}[\mathbb{I}, \text{ROW}(3)]$. Handle edge effects by replication. Show the result "image" $\mathbb{J}$.

$$\mathbb{J} = \text{DILATE}\{[1\,2\,4\,1\,3],\ \text{ROW}(3)\}$$

$$= \text{MAX}_3\,[1\,\vdots\,1\,2\,4\,1\,3\,\vdots\,3] = \underline{\underline{[2\ 4\ 4\ 4\ 3]}}$$

(b) **14 pts.** Now obtain the same result by implementing the DILATION filter as a stack filter. **Hint:** compute the binary signals that make up the threshold decomposition of $\mathbb{I}$, apply a binary DILATION filter to each threshold signal, and then combine the results using the stacking property to get the final gray scale answer.

	1	1	2	4	1	3	3	BINARY 3-PT MAX				
≥ 4	0	0	0	1	0	0	0	0	1	1	1	0
≥ 3	0	0	0	1	0	1	1	0	1	1	1	1
≥ 2	0	0	1	1	0	1	1	1	1	1	1	1
≥ 1	1	1	1	1	1	1	1	1	1	1	1	1
≥ 0	1	1	1	1	1	1	1	1	1	1	1	1

$\underline{\underline{[2\ 4\ 4\ 4\ 3]}}$

5. **20 pts.** Consider the *cameraman* image $\mathbb{I}$ shown below.

The size of the image is 256×256 pixels and each pixel has eight bits. Five grayscale morphological filters are applied, all with respect to the structuring element $\mathbb{B} =$ SQUARE(9), to define five new filtered images according to

$$\begin{aligned} \mathbb{J}_M &= \mathrm{MED}(\mathbb{I}, \mathbb{B}), \\ \mathbb{J}_E &= \mathrm{ERODE}(\mathbb{I}, \mathbb{B}), \\ \mathbb{J}_D &= \mathrm{DILATE}(\mathbb{I}, \mathbb{B}), \\ \mathbb{J}_O &= \mathrm{OPEN}(\mathbb{I}, \mathbb{B}), \\ \mathbb{J}_C &= \mathrm{CLOSE}(\mathbb{I}, \mathbb{B}). \end{aligned}$$

Label the five output images shown on the next page.

Problem 5 cont...

OPEN
(small bright objects removed, small dark objects preserved)

MEDIAN
(small objects removed, both light and dark)

CLOSE
(small bright objects preserved, small dark objects removed)

DILATE
(the brightest)

ERODE
(the darkest)