ECE 5273
Test 2

Tuesday, May 7, 2013
10:30 AM - 12:30 PM

Spring 2013

Dr. Havlicek

Name: SOLUTION

Student Num:________________

Directions: This is an open notes test. You may use a clean copy of the course notes as published on the course web site. Other materials are not allowed. You have 120 minutes to complete the test. All work must be your own.

SHOW ALL OF YOUR WORK for maximum partial credit!

GOOD LUCK!

SCORE:

1. (20) ________
2. (20) ________
3. (20) ________
4. (20) ________
5. (20) ________

TOTAL (100):

____

On my honor, I affirm that I have neither given nor received inappropriate aid in the completion of this test.

Name:________________ Date:________________

1. **20 pts**. True or False. Mark *True* only if the statement is **always** true.

TRUE	FALSE	
X		(a) **2 pts**. If you want to implement the **linear** convolution of two 256 × 256 digital images by pointwise multiplication of DFT's, then it is generally necessary to zero pad the images to a size of 512 × 512 pixels.
	X	(b) **2 pts**. Median filtering is almost always always implemented by pointwise multiplication of DFT's. NONLINEAR
	X	(c) **2 pts**. The median filter removes positive impulses while preserving negative impulses. Removes both
	X	(d) **2 pts**. Streaking and blotching often occur when the OPEN filter is applied with a window that is too small. LARGE
	X	(e) **2 pts**. The frequency response of the CLOSE-OPEN filter exhibits a "low-pass" characteristic. NOT LINEAR
	X	(f) **2 pts**. The inverse filter is optimal for performing deconvolution when there is also additive white noise. Only when no additive noise
X		(g) **2 pts**. In the classical linear image restoration problem, the Wiener filter reduces to the inverse filter if there is no additive noise.
X		(h) **2 pts**. An advantage of the LoG edge detector is that edge thinning and edge linking are not required.
	X	(i) **2 pts**. The Sobel edge detector is the best choice for detecting edges in noisy images. LoG is better
	X	(j) **2 pts**. The "Wiener filter" was invented by U.S. Congressional Representative Anthony Weiner, who used it to process the images he sent using Twitter.

2. **20 pts**. For the window (structuring element) SQUARE(9), give the order statistic (OS) filter weights (coefficients) $\mathbf{A}^T$ for

(a) **4 pts.** A median filter:

$\mathbf{A}^T = [0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0]$

(b) **4 pts.** An average filter:

$\mathbf{A}^T = [\frac{1}{9}\ \frac{1}{9}\ \frac{1}{9}\ \frac{1}{9}\ \frac{1}{9}\ \frac{1}{9}\ \frac{1}{9}\ \frac{1}{9}\ \frac{1}{9}]$

(c) **4 pts.** An OS filter to perform morphological erosion:

$\mathbf{A}^T = [1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]$

(d) **4 pts.** An OS filter to perform morphological dilation:

$\mathbf{A}^T = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1]$

(e) **4 pts.** The inner average OS filter INNER_AVE$_2$:

$P = 2$

$\mathbf{A}^T = [0\ 0\ \frac{1}{5}\ \frac{1}{5}\ \frac{1}{5}\ \frac{1}{5}\ \frac{1}{5}\ 0\ 0]$

3. **20 pts**. The $N \times N$ digital image $\mathbb{I}_1$ is defined by

$$I_1(i,j) = 5 + 2\delta(i-1, j-4)$$

and the $N \times N$ digital image $\mathbb{I}_2$ is defined by

$$I_2(i,j) = 6 + \cos\left[\frac{2\pi}{N}(13i - 7j)\right].$$

Let the $N \times N$ digital image $\mathbb{J}$ be the wraparound convolution of $\mathbb{I}_1$ and $\mathbb{I}_2$. Find $\widetilde{\mathbb{J}}$, the DFT of $\mathbb{J}$.

NOTES p. 4.41: $\quad \mathrm{DFT}[5] = 5N\delta(u,v) \qquad \mathrm{DFT}[6] = 6N\delta(u,v)$

NOTES p. 4.16: $\quad \mathrm{DFT}[2\delta(i-1,j-4)] = \frac{1}{N}\sum_{i=0}^{N-1}\sum_{j=0}^{N-1} 2\delta(i-1,j-4)\, W_N^{(ui+vj)}$

one nonzero term, when $i=1$ and $j=4$

$$= \frac{1}{N}\cdot 2 \cdot W_N^{(u+4v)}$$

$$= \frac{2}{N} W_N^{(u+4v)} = \frac{2}{N}\left[e^{-\sqrt{-1}\frac{2\pi}{N}}\right]^{u+4v} = \frac{2}{N} e^{-\sqrt{-1}\,2\pi(u+4v)/N}$$

NOTES p. 4.42: $\quad \mathrm{DFT}\left\{\cos\left[\frac{2\pi}{N}(13i-7j)\right]\right\} = \frac{N}{2}\left[\delta(u-13,v+7) + \delta(u+13,v-7)\right]$

$$\widetilde{\mathbb{I}}_1 = \mathrm{DFT}[5] + \mathrm{DFT}[2\delta(i-1,j-4)] = 5N\delta(u,v) + \frac{2}{N} W_N^{(u+4v)}$$

$$\widetilde{\mathbb{I}}_2 = \mathrm{DFT}[6] + \mathrm{DFT}\left\{\cos\left[\frac{2\pi}{N}(13i-7j)\right]\right\} = 6N\delta(u,v) + \frac{N}{2}\left[\delta(u-13,v+7) + \delta(u+13,v-7)\right]$$

NOTES p. 5.4:

$$\widetilde{\mathbb{J}} = N\widetilde{\mathbb{I}}_1 \otimes \widetilde{\mathbb{I}}_2 = \left[5N^2\delta(u,v) + 2W_N^{(u+4v)}\right]\widetilde{\mathbb{I}}_2 = 5N^2\delta(u,v)\widetilde{\mathbb{I}}_2 + 2W_N^{(u+4v)}\widetilde{\mathbb{I}}_2$$

$$= 5N^2\delta(u,v)\left\{6N\delta(u,v) + \frac{N}{2}\left[\delta(u-13,v+7) + \delta(u+13,v-7)\right]\right\}$$

$$+ 2W_N^{(u+4v)}\left\{6N\delta(u,v) + \frac{N}{2}\left[\delta(u-13,v+7) + \delta(u+13,v-7)\right]\right\}$$

$$= 30N^3\delta^2(u,v) + \frac{5N^3}{2}\left[\overbrace{\delta(u,v)\delta(u-13,v+7)}^{\text{ZERO}} + \overbrace{\delta(u,v)\delta(u+13,v-7)}^{\text{ZERO}}\right]$$

$$+ 12N\delta(u,v)W_N^{(u+4v)} + N\delta(u-13,v+7)W_N^{(u+4v)} + N\delta(u+13,v-7)W_N^{(u+4v)}$$

$$= 30N^3\delta(u,v) + 12N\delta(u,v)\underbrace{W_N^0}_{\text{ONE}} + N\delta(u-13,v+7)W_N^{(13-28)} + N\delta(u+13,v-7)W_N^{(-13+28)}$$

$$= 30N^3\delta(u,v) + 12N\delta(u,v) + N\delta(u-13,v+7)W_N^{-15} + N\delta(u+13,v-7)W_N^{15}$$

$$\boxed{\widetilde{\mathbb{J}} = (30N^3 + 12N)\delta(u,v) + N\delta(u-13,v+7)W_N^{-15} + N\delta(u+13,v-7)W_N^{15}}$$

4. **20 pts**. Pixels in the 4×4 image $\mathbb{I}$ shown below take values in the range $0 \le I(i, j) \le 99$. The image is transmitted through a communication channel where it is corrupted by noise. The received image $\mathbb{J}$ is also shown below.

$\mathbb{I}$ =

72	**72**	**73**	**74**
72	**99**	**72**	**74**
74	**75**	**71**	**70**
75	**71**	**69**	**69**

$\mathbb{J}$ =

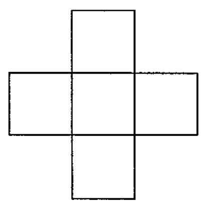
72	**72**	**73**	**74**
72	**99**	**72**	**74**
74	**75**	**0**	**70**
75	**71**	**69**	**69**

Choose an appropriate morphological operation (MAJ, MED, ERODE, DILATE, OPEN, CLOSE) to restore the received image by attenuating the transmission noise and compute the ISNR.

Use the structuring element $\mathbb{B}$ = CROSS(5) shown below:

NOTE: This solution replicates the edge effects before each step. That is okay, but not as good as the following alternate solution - - -

$\mathbb{B}$ =



Justify your choice for the operation and show the restored image $\widehat{\mathbb{I}}$ below. Handle edge effects by replication. Work space is provided on the following page.

- The noise introduces a negative impulse → use MEDIAN or CLOSE
- The image contains a positive impulse that should be preserved

⟹ USE CLOSE

Show the restored image here:

$\widehat{\mathbb{I}}$ =

72	72	74	74
72	99	74	74
75	75	71	70
75	71	70	70

ISNR = 27.9943 dB

Work Space for Problem 4...

$$MSE(\mathbb{J}) = \frac{1}{N^2}\sum_{i=0}^{3}\sum_{j=0}^{3}\left[J(i,j)-I(i,j)\right]^2 = \frac{71^2}{16} = \frac{5041}{16}$$

$\mathbb{J}-\mathbb{I}$

0	0	0	0
0	0	0	0
0	0	-71	0
0	0	0	0

$$MSE(\hat{\mathbb{I}}) = \frac{1}{N^2}\sum_{i=0}^{3}\sum_{j=0}^{3}\left[\hat{I}(i,j)-I(i,j)^2\right] = \frac{8}{16}$$

$$ISNR = 10\log_{10}\frac{MSE(\mathbb{J})}{MSE(\hat{\mathbb{I}})} = 10\log_{10}\frac{5041/16}{8/16} = 10\log_{10}\frac{5041}{8}$$

$$= 27.9943 \text{ dB}$$

$\mathbb{J} =$

	72	72	73	74	
72	72	72	73	74	74
72	72	99	72	74	74
74	74	75	0	70	70
75	75	71	69	69	69
	75	71	69	69	

$\mathbb{K} = \text{DILATE}[\mathbb{J}]$

	72	99	74	74	
72	72	99	74	74	74
99	99	99	99	74	74
75	75	99	75	74	74
75	75	75	71	70	70
	75	75	71	70	

$\hat{\mathbb{I}} = \text{ERODE}[\mathbb{K}]$

72	72	74	74
72	99	74	74
75	75	71	70
75	71	70	70

$\hat{\mathbb{I}} - \mathbb{I}$

0	0	1	0
0	0	2	0
1	0	0	0
0	0	1	1

ALTERNATE SOLUTION

4. **20 pts**. Pixels in the 4×4 image $\mathbb{I}$ shown below take values in the range $0 \leq I(i,j) \leq 99$. The image is transmitted through a communication channel where it is corrupted by noise. The received image $\mathbb{J}$ is also shown below.

$\mathbb{I}$ =

72	**72**	**73**	**74**
72	**99**	**72**	**74**
74	**75**	**71**	**70**
75	**71**	**69**	**69**

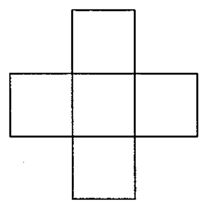
$\mathbb{J}$ =

72	**72**	**73**	**74**
72	**99**	**72**	**74**
74	**75**	**0**	**70**
75	**71**	**69**	**69**

Choose an appropriate morphological operation (MAJ, MED, ERODE, DILATE, OPEN, CLOSE) to restore the received image by attenuating the transmission noise and compute the ISNR.

Use the structuring element $\mathbb{B}$ = CROSS(5) shown below:

$\mathbb{B}$ =



Justify your choice for the operation and show the restored image $\widehat{\mathbb{I}}$ below. Handle edge effects by replication. Work space is provided on the following page.

AS in the previous solution, you should use CLOSE. This time we will do all the edge replication up front at the very beginning. In this case, it will give us a little bit better solution, although that does not always happen

Show the restored image here:

$\widehat{\mathbb{I}}$ =

72	72	74	74
72	99	74	74
75	75	71	70
75	71	70	69

ISNR = 28.5742dB

Work Space for Problem 4...

As before in the previous solution, $MSE(\mathbb{J}) = \frac{5041}{16}$

But this time,

$$MSE(\hat{\mathbb{I}}) = \frac{1}{N^2} \sum_{i=0}^{3} \sum_{j=0}^{3} \left[\hat{I}(i,j) - I(i,j) \right]^2 = \frac{7}{16}$$

$$ISNR = 10 \log_{10} \frac{MSE(\mathbb{J})}{MSE(\hat{\mathbb{I}})} = 10 \log_{10} \frac{5041/16}{7/16}$$

$$= 10 \log_{10} \frac{5041}{7} = 28.5742 \text{ dB}$$

$\mathbb{K} = \text{DILATE}[\mathbb{J}]$

$\mathbb{J} =$

	72	72	72	73	74	74	
72	72	72	72	73	74	74	
72	72	72	72	73	74	74	74
72	72	72	99	72	74	74	74
74	74	74	75	0	70	70	70
75	75	75	71	69	69	69	69
75	75	75	71	69	69	69	69
	75	75	71	69	69	69	

$\mathbb{K}$:

	72	73	74	74	
72	72	99	74	74	74
74	99	99	99	74	74
75	75	99	75	74	74
75	75	75	71	70	70
	75	75	71	69	

$\hat{\mathbb{I}} = \text{ERODE}[\mathbb{K}]$

72	72	74	74
72	99	74	74
75	75	71	70
75	71	70	69

$\hat{\mathbb{I}} - \mathbb{I}$

0	0	1	0
0	0	2	0
1	0	0	0
0	0	1	0

5. **20 pts.** The Blowfish Digital Photographic Services Corporation makes money by scanning customer photographs to convert them into gray scale digital images.

Similar to a photocopier, the scanner contains a linear array sensor with 5100 detectors (pixels). The sensor is 8.5" long and has 600 detectors per inch.

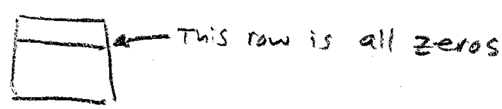
Thus a 'snapshot' from this sensor produces one column of the digital image and the sensor is mechanically scanned from left to right while taking successive 'snapshots' to form the entire scanned digital image.

Unfortunately, the 1026'th detector burned out recently. The result is that this failed detector always outputs the minimum pixel value of zero. Therefore, the 1026'th row of every digital image obtained from this scanner is all zeros.

Because it would be too expensive to replace the scanner, you have been hired to fix the problem with digital image processing.

You can think of this as a noise removal problem where the black row of dead pixels is considered to be due to additive noise. Design a gray scale order statistic filter to enhance the scanned digital images by reducing this noise. Be sure to indicate which filtering operation you recommend as well as the window. Explain your answer.

- The result of the burned out detector is a row of zero pixels or "dead" pixels:



- The window should be as small as possible so that the "good" pixels are changed as little as possible.

⇒ Interpolate each "dead" pixel using the good pixel above and the good pixel below.

⇒ The window should be COL(3) =  .

- Within this window, each dead pixel looks like a negative spike.

⇒ USE MEDIAN

⇒ USE A 3-PT MEDIAN FILTER WITH WINDOW COL(3)