ECE 5273
Test 2

Monday, May 4, 2015
10:30 AM - 12:30 PM

Spring 2015

Dr. Havlicek

Name: SOLUTION

Student Num: ____________________

Directions: This is an open notes test. You may use a clean copy of the course notes as published on the course web site. Other materials are not allowed. You have 120 minutes to complete the test. All work must be your own.

SHOW ALL OF YOUR WORK for maximum partial credit!

GOOD LUCK!

SCORE:

1. (20) ________
2. (20) ________
3. (20) ________
4. (20) ________
5. (20) ________

TOTAL (100):

____

On my honor, I affirm that I have neither given nor received inappropriate aid in the completion of this test.

Name: ____________________ Date: ____________________

1. **20 pts**. True or False. Mark *True* only if the statement is **always** true.

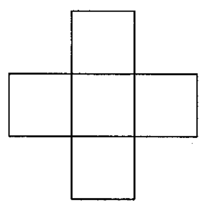
TRUE	FALSE	
	X	(a) **2 pts**. The main reason that the AVERAGE, MEDIAN, CLOSE-OPEN, and OPEN-CLOSE filters are all good for denoising is that they all have low pass frequency responses. ← Nonlinear = no freq. resp.
X		(b) **2 pts**. The reason that differential pulse code modulation (DPCM) is applied to the discrete cosine transform (DCT) DC coefficients from each block in baseline JPEG is to reduce entropy. Notes p. 7.79
X		(c) **2 pts**. For a grayscale image that is not constant, repeatedly applying a DILATION filter will reduce the entropy. Creates constant blobs & finally a constant image.
X		(d) **2 pts**. In order to compute the *linear* convolution of two 256×256 grayscale images by pointwise multiplication of DFT's, both images must be zero padded to a *minimum* size of 511×511. Notes p. 5.44
X		(e) **2 pts**. In image processing, high pass filters are typically used to enhance details and to remove blur. Notes p. 5.94
	X	(f) **2 pts**. The main advantage of the Laplacian-of-Gaussian (LoG) edge detector is that the Laplacian operator is less noise sensitive than the derivative operator found in the gradient-based edge detectors. It's more sensitive. Notes p. 8.72
	X	(g) **2 pts**. The main difference between the grayscale median and OPEN-CLOSE filters is that the OPEN-CLOSE filter does not eradicate negative impulses. It eradicates both. Notes p. 6.37
	X	(h) **2 pts**. In grayscale template matching, the maximum value of the cross-correlation between the template and the image patch that it covers is given by the square of the template energy. Notes p. 8.38-8.42
X		(i) **2 pts**. Best paper awards at the 2015 IEEE International Conference on Image Processing (ICIP) will be presented by Lena Söderberg, subject of the famous "Lenna" image.
Ridiculous!		(j) **2 pts**. Although homomorphic filtering has been legalized in 37 states and the District of Columbia, it is still against the law in Oklahoma.

2. **20 pts.** The 4×4 image $\mathbf{I}$ shown on the left below has 8-bit pixels in the range $0 \leq I(m,n) \leq 255$. This image is transmitted through a communication channel where it is corrupted by IID additive white Laplacian noise with parameter $\sigma = 4$. The received image $\mathbf{J}$ is shown on the right below.

$$\mathbf{I} = \begin{array}{|c|c|c|c|}\hline 32 & 32 & 32 & 32 \\ \hline 120 & 120 & 32 & 32 \\ \hline 120 & 120 & 120 & 32 \\ \hline 120 & 120 & 120 & 120 \\ \hline \end{array} \qquad \mathbf{J} = \begin{array}{c|c|c|c|c|c} & 30 & 32 & 28 & 35 & \\ \hline 30 & 30 & 32 & 28 & 35 & 35 \\ \hline 125 & 125 & 134 & 39 & 36 & 36 \\ \hline 117 & 117 & 115 & 107 & 27 & 27 \\ \hline 123 & 123 & 120 & 122 & 119 & 119 \\ \hline & 123 & 120 & 122 & 119 & \end{array}$$

(a) **15 pts.** Choose the best OS or morphological filter (MED, AVE, INNER_AVE, ERODE, DILATE, OPEN, CLOSE, *etc.*) to denoise the received image by attenuating the transmission noise. Use the structuring element/window $\mathbf{B} = \text{CROSS}(5)$ shown below. **Show** the denoised image $\widehat{\mathbf{I}}$ in the space provided at the bottom of this page. Handle edge effects by replication.

Notes p. 6.68:
MEDIAN is best for Laplacian noise.

$\mathbf{B} =$ 

Note: Work space is provided on the following page.

(b) **5 pts.** Compute the ISNR for the denoised image.

ISNR is defined on p. 5.115 of the notes.

$$\text{ISNR}(\hat{I}) = 10 \log_{10} \frac{\text{MSE}(J)}{\text{MSE}(\hat{I})} = 10 \log_{10} \frac{557/16}{341/16} = 10 \log_{10} \frac{557}{341}$$

$$\approx 10 \log_{10} 1.6334 \approx 2.1310 \text{ dB}$$

Show the denoised image here:

$$\widehat{\mathbf{I}} = \begin{array}{|c|c|c|c|}\hline 30 & 32 & 32 & 35 \\ \hline 125 & 115 & 39 & 36 \\ \hline 117 & 117 & 107 & 36 \\ \hline 123 & 120 & 120 & 119 \\ \hline \end{array}$$

ISNR = 2.1310 dB

Work Space for Problem 2...

MSE is defined on page 5.114 of the notes.

$$MSE(J) = \frac{1}{16}\sum (J-I)^2 = \frac{1}{16}\cdot 557 \approx 34.8125$$

$$MSE(\hat{I}) = \frac{1}{16}\sum (\hat{I}-I)^2 = \frac{1}{16}\cdot 341 \approx 21.3125$$

$J - I \approx$

-2	0	-4	3
5	14	7	4
-3	-5	-13	-5
3	0	2	-1

$(J-I)^2 =$

4	0	16	9
25	196	49	16
9	25	169	25
9	0	4	1

$\hat{I} - I =$

-2	0	0	3
5	-5	7	4
-3	-3	-13	4
3	0	0	-1

$(\hat{I}-I)^2 =$

4	0	0	9
25	25	49	16
9	9	169	16
9	0	0	1

3. **20 pts.** The grayscale image $\mathbf{I}$ has 4-bit pixels in the range $0 \leq I(m,n) \leq 15$. A grayscale median filter is applied with a window that covers seven pixels. For a certain input pixel, the window set is given by $\mathbf{B} \circ \mathbf{I}(i,j) = \{1,\ 7,\ 15,\ 5,\ 9,\ 3,\ 12\}$. Apply Delman's bit-serial median filtering algorithm to find the median.

Delman's algorithm is described on pages 6.55 – 6.58 of the notes.

Original Data:

$$\begin{array}{cccc} 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{array}$$

Majority: 0 - - -

First Change:

$$\begin{array}{cccc} 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{array}$$

Majority: 0 1 - -

Second Change:

$$\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{array}$$

Majority: 0 1 1 _

Third Change:

$$\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{array}$$

Majority: 0 1 1 1

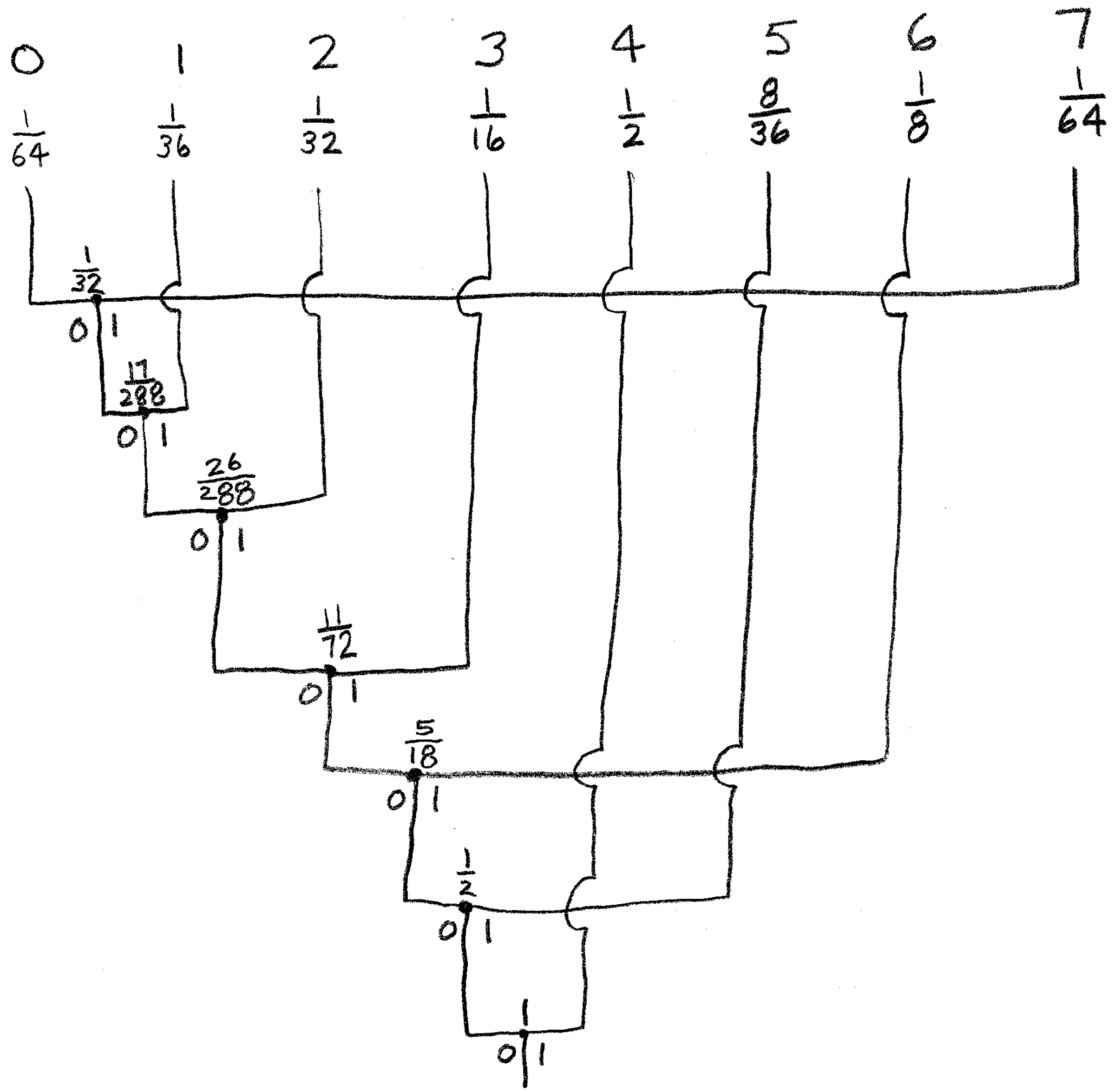
MEDIAN = 0111 binary = $\underline{\underline{7}}$

4. **20 pts.** Gray scale digital images **I** with 3 bits per pixel and gray levels in the range $\{0, 1, \ldots, 7\}$ are modeled as coming from an information source with the following source symbol probabilities (normalized histogram):

Huffman coding is explained in the notes on pages 7.24 – 7.29

k	0	1	2	3	4	5	6	7
$p_{\mathbf{I}}(k)$	$\frac{1}{64}$	$\frac{1}{36}$	$\frac{1}{32}$	$\frac{1}{16}$	$\frac{1}{2}$	$\frac{8}{36}$	$\frac{1}{8}$	$\frac{1}{64}$

(a) **12 pts.** Design a Huffman code to encode these images.



K	0	1	2	3	4	5	6	7
C(k)	0000000	000001	00001	0001	1	01	001	0000001
L[C(k)]	7	6	5	4	1	2	3	7

Problem 4 cont...

(b) **4 pts.** Find the expected BPP (bits per pixel) and CR (compression ratio) for the coded images $C(\mathbf{I})$.

→ The calculation for BPP of the coded image is on notes p. 7.24.

$$\text{Coded BPP} = \sum_{k=0}^{7} p_I(k)\, L[c(k)]$$

$$= \tfrac{1}{64}\cdot 7 + \tfrac{1}{36}\cdot 6 + \tfrac{1}{32}\cdot 5 + \tfrac{1}{16}\cdot 4 + \tfrac{1}{2}\cdot 1 + \tfrac{8}{36}\cdot 2 + \tfrac{1}{8}\cdot 3 + \tfrac{1}{64}\cdot 7$$

$$= \tfrac{7}{64} + \tfrac{6}{36} + \tfrac{5}{32} + \tfrac{4}{16} + \tfrac{1}{2} + \tfrac{16}{36} + \tfrac{3}{8} + \tfrac{7}{64}$$

$$= \frac{7}{2^6} + \frac{6}{2^2\cdot 3^2} + \frac{5}{2^5} + \frac{4}{2^4} + \frac{1}{2} + \frac{16}{2^2\cdot 3^2} + \frac{3}{2^3} + \frac{7}{2^6}$$

$$= \frac{7\cdot 3^2 + 6\cdot 2^4 + 5\cdot 2\cdot 3^2 + 4\cdot 2^2\cdot 3^2 + 2^5\cdot 3^2 + 16\cdot 2^4 + 3\cdot 2^3\cdot 3^2 + 7\cdot 3^2}{2^6\cdot 3^2}$$

$$= \frac{63 + 96 + 90 + 144 + 288 + 256 + 216 + 63}{576} = \frac{1{,}216}{576} = \frac{19}{9} \approx 2.1111$$

$$\boxed{\text{BPP} = \tfrac{19}{9} \approx 2.1111}$$

→ The calculation for the CR is given on notes p. 7.8.

$$\text{CR} = \frac{\text{original BPP}}{\text{coded BPP}} = \frac{3}{1{,}216/576} = \frac{576\cdot 3}{1{,}216} = \frac{9\cdot 3}{19} = \frac{27}{19} \approx 1.4211$$

(c) **4 pts.** Does your code achieve the theoretical lower bound on BPP for the coded images? Explain why or why not.

NO. The symbol probabilities for $k=1$ and $k=5$ are not integer powers of 2. (See notes p. 7.23)

5. **20 pts.** Consider the *cameraman* image $\mathbb{I}$ shown below.

The size of the image is 256×256 pixels and each pixel has eight bits. Five grayscale morphological filters are applied, all with respect to the structuring element $\mathbb{B} =$ SQUARE(9), to define five new filtered images according to

$$\begin{aligned}
\mathbb{J}_M &= \text{MED}(\mathbb{I}, \mathbb{B}), \\
\mathbb{J}_E &= \text{ERODE}(\mathbb{I}, \mathbb{B}), \\
\mathbb{J}_D &= \text{DILATE}(\mathbb{I}, \mathbb{B}), \\
\mathbb{J}_O &= \text{OPEN}(\mathbb{I}, \mathbb{B}), \\
\mathbb{J}_C &= \text{CLOSE}(\mathbb{I}, \mathbb{B}).
\end{aligned}$$

Label the five output images shown on the next page.

Problem 5 cont...

MEDIAN

(small objects removed, both light and dark)

OPEN

(small bright objects removed, small dark objects preserved)

CLOSE

(small bright objects preserved, small dark objects removed)

DILATE

(the brightest)

ERODE

(the darkest)