ECE 5273
Test 2

Wednesday, May 2, 2018
4:30 PM - 6:30 PM

Spring 2018

Dr. Havlicek

Name: SOLUTION

Student Num:__________

Directions: This is an open notes test. You may use a clean copy of the course notes as published on the course web site. Other materials are not allowed. You have 120 minutes to complete the test. All work must be your own.

SHOW ALL OF YOUR WORK for maximum partial credit!

GOOD LUCK!

SCORE:

1. (25) ________

2. (25) ________

3. (25) ________

4. (25) ________

TOTAL (100):

On my honor, I affirm that I have neither given nor received inappropriate aid in the completion of this test.

Name:__________ Date:__________

1. **25 pts.** True or False. Mark *True* only if the statement is **always** true.

TRUE	FALSE	
X		(a) **2 pts.** If two images are discrete and periodic with the same period, then their wraparound convolution equals their linear convolution.
X		(b) **2 pts.** With appropriate zero padding, wraparound convolution can be used to implement linear convolution.
	X	(c) **2 pts.** The average filter AVE[**J**, **B**] is an example of a nonlinear filter.
	X	(d) **2 pts.** Low-pass filters are typically used to enhance image details and contrast. NOTES P. 5-94
	X	(e) **2 pts.** The Laplacian filter is an important example of a band-pass filter. Notes p. 5-99
	X	(f) **2 pts.** When the spectra of the noise and the image are overlapping, the window of a *linear* denoising filter must be designed very carefully in order to ensure that *all* of the noise is removed. Notes p. 5-106
X		(g) **2 pts.** For a denoising filter, higher *ISNR* means better performance. Notes p. 5-105.
	X	(h) **2 pts.** In practical image restoration problems (linear blur plus additive noise), the blur is almost always high-pass. Notes p. 5-118
X		(i) **2 pts.** A filter is **robust** if it gives close to optimal performance for a wide variety of noise types. Notes p. 6-70
X		(j) **2 pts.** An image with a flat histogram cannot be compressed by a variable wordlength code alone. Notes p. 7-18
X		(k) **2 pts.** The JPEG baseline algorithm applies run-length coding to the quantized and zig-zag ordered AC coefficients while applying DPCM to the quantized DC coefficients. Notes p. 7-79, 7-81
OH	MY!	(l) **3 pts.** The Wiener filter was first introduced in 1945 by famous MIT electrical engineer Oscar F. Mayer.

2. **25 pts.** The $N \times N$ digital image $\mathbf{I}_1$ is defined by

$$I_1(m,n) = 5 + 3\delta(m,n)$$

and the $N \times N$ digital image $\mathbf{I}_2$ is defined by

$$I_2(m,n) = 1 + 6\cos\left[\frac{2\pi}{N}(2m+7n)\right].$$

Let the $N \times N$ digital image $\mathbf{J}$ be the wraparound convolution of $\mathbf{I}_1$ and $\mathbf{I}_2$. Find $\widetilde{\mathbf{J}}$, the DFT of $\mathbf{J}$.

NOTES p. 4.126: $5 \leftrightarrow 5N^2\delta(u,v)$; $1 \leftrightarrow N^2\delta(u,v)$

p. 4.127: $3\delta(m,n) \leftrightarrow 3$

p. 4.128: $6\cos\left[2\pi\left(\frac{2}{N}m + \frac{7}{N}n\right)\right] \leftrightarrow 3N^2\left[\delta(u-2,v-7) + \delta(u+2,u+7)\right]$

$$\tilde{I}_1 = 5N^2\delta(u,v) + 3$$

$$\tilde{I}_2 = N^2\delta(u,v) + 3N^2\left[\delta(u-2,v-7) + \delta(u+2,v+7)\right]$$

NOTES p. 5-15, 5-26:

$$\tilde{J} = \tilde{I}_1 \otimes \tilde{I}_2 = \left\{5N^2\delta(u,v)+3\right\}\left\{N^2\delta(u,v) + 3N^2\left[\delta(u-2,v-7)+\delta(u+2,v+7)\right]\right\}$$

$$= 5N^4\delta(u,v) + \underbrace{15N^4\delta(u,v)\left[\delta(u-2,v-7)+\delta(u+2,v+7)\right]}_{\text{zero}} + 3N^2\delta(u,v) + 9N^2\left[\delta(u-2,v-7)+\delta(u+2,v+7)\right]$$

$$= \left[5N^4+3N^2\right]\delta(u,v) + 9N^2\left[\delta(u-2,v-7)+\delta(u+2,v+7)\right]$$

$$\boxed{\tilde{J} = \left[5N^4+3N^2\right]\delta(u,v) + 9N^2\left[\delta(u-2,v-7)+\delta(u+2,v+7)\right]}$$

Work Space for Problem 2...

3. **25 pts.** Pixels in the 6×6 image **I** shown below take values in the range $\{0, 1, 2, \ldots, 127\}$. The image is sent through a communication channel where it is corrupted by additive noise. The received image **J** is shown below.

I =

72	**72**	**72**	**10**	**10**	**10**
72	**72**	**72**	**10**	**10**	**10**
72	**72**	**72**	**10**	**10**	**10**
72	**72**	**72**	**10**	**10**	**10**
72	**72**	**72**	**10**	**10**	**10**
72	**72**	**72**	**10**	**10**	**10**

J =

72	**72**	**72**	**10**	**10**	**10**
72	**72**	**72**	**10**	**10**	**10**
72	**72**	**65**	**10**	**44**	**10**
72	**72**	**71**	**53**	**10**	**10**
72	**72**	**72**	**10**	**10**	**10**
72	**72**	**72**	**10**	**10**	**10**

(a) **18 pts.** Design a nonlinear filter to denoise the received image **J**. Explain the rationale of your design. Show the denoised image **K** in the space provided below.

(b) **7 pts.** Find the *ISNR* of your denoised image **K**.

Workspace is given on the next two pages.

Since the noise is mostly impulsive and we need to preserve the edge in I, the median filter will work well. In general, we want the smallest possible window that will remove the noise, since this will avoid changing the pixels that were not corrupted. In this case, B = COL(5) will do the trick. For this J image, SQUARE(25) will also produce the same result. So my filter is K = MED[J, B] where B = COL(5).

K =

72	72	72	10	10	10
72	72	72	10	10	10
72	72	72	10	10	10
72	72	72	10	10	10
72	72	72	10	10	10
72	72	72	10	10	10

See work on Next Page

ISNR(K) = ∞ dB

Handle edge effects by replication

$K = \text{MED}[J, \text{COL}(5)]$

Workspace for Problem 3...

$J =$

72	72	72	10	10	10
72	72	72	10	10	10
72	72	65	10	44	10
72	72	71	53	10	10
72	72	72	10	10	10
72	72	72	10	10	10

$K =$

72	72	72	10	10	10
72	72	72	10	10	10
72	72	72	10	10	10
72	72	72	10	10	10
72	72	72	10	10	10
72	72	72	10	10	10

ISNR is defined on Notes p. 5-115

$I =$

72	72	72	10	10	10
72	72	72	10	10	10
72	72	72	10	10	10
72	72	72	10	10	10
72	72	72	10	10	10
72	72	72	10	10	10

$K - I$

0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

$$\text{MSE}(K) = \frac{0}{36} = 0$$

$J - I$

0	0	0	0	0	0
0	0	0	0	0	0
0	0	-7	0	34	0
0	0	-1	43	0	0
0	0	0	0	0	0
0	0	0	0	0	0

$$\text{ISNR}(K) = 10\log_{10}\frac{\text{MSE}(J)}{\text{MSE}(K)}$$

$$= \lim_{\varepsilon \to 0} 10\log_{10}\frac{84.86}{\varepsilon}$$

$$= 10\log_{10}\lim_{x \to \infty} x$$

$$= \underline{\underline{\infty \text{ dB}}}$$

$$\text{MSE}(J) = \frac{(-7)^2 + (-1)^2 + (43)^2 + (34)^2}{36}$$

$$= \frac{49 + 1 + 1849 + 1156}{36}$$

$$= \frac{3055}{36} = 84.86$$

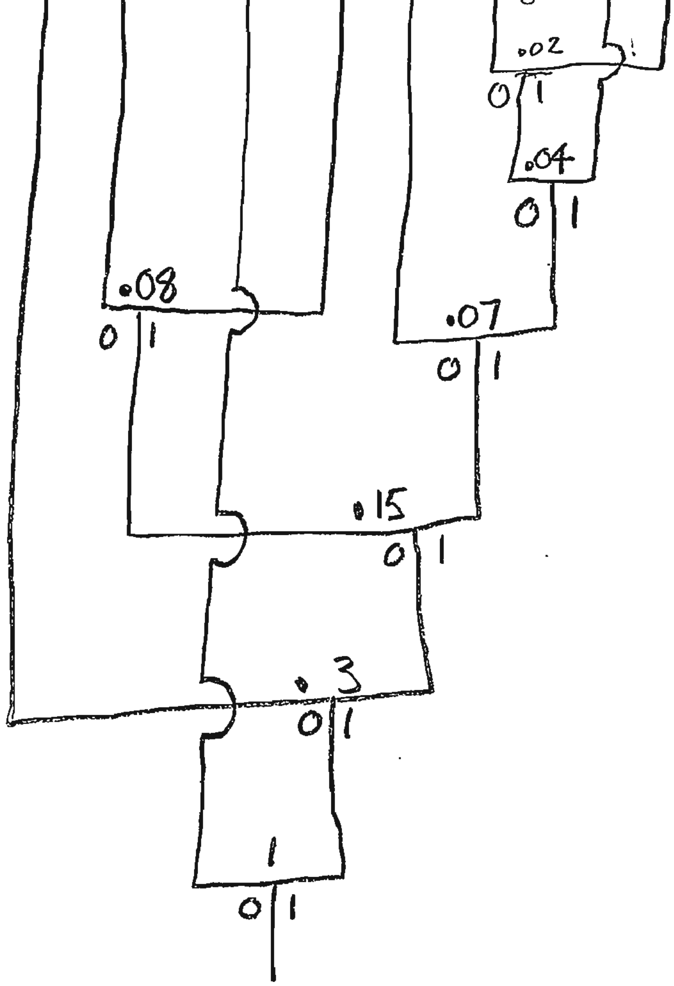
Workspace for Problem 3...

4. **25 pts.** Gray scale digital images **I** with 3 bits per pixel and gray levels in the range $\{0, 1, \ldots, 7\}$ are modeled as coming from an information source with the following source symbol probabilities (normalized histogram):

k	0	1	2	3	4	5	6	7
$p_I(k)$	0.15	0.04	0.70	0.04	0.03	0.01	0.02	0.01

(a) **15 pts.** Design a Huffman code to encode these images.

Notes
p. 7-27



k	0	1	2	3	4	5	6	7
C(k)	10	1100	0	1101	1110	111100	11111	111101
L[C(k)]	2	4	1	4	4	6	5	6

Problem 4 cont...

(b) **5 pts.** Find the expected BPP (bits per pixel) and CR (compression ratio) for the coded images $C(\mathbf{I})$.

Original BPP = 3

$$\text{New BPP} = \overset{-0-}{(0.15)(2)} + \overset{-1-}{(0.04)4} + \overset{-2-}{(0.7)(1)} + \overset{-3-}{(0.04)(4)} + \overset{-4-}{(0.03)(4)}$$
$$+ \overset{-5-}{(0.01)(6)} + \overset{-6-}{(0.02)(5)} + \overset{-7-}{(0.01)(6)}$$

$= 1.66$

$$\boxed{BPP = 1.66}$$

$$\boxed{CR = \frac{3}{1.66} = 1.8072 : 1}$$

(c) **5 pts.** Does your code achieve the theoretical lower bound on BPP for the coded images? Explain why or why not.

<u>NO</u>. Because some of the symbol probabilities are not integer powers of 2.

For example, $P(2) = 0.7$, which is not an integer power of 2.