# ECE 5273
# Test 2

Wednesday, May 8, 2019
4:30 PM - 6:30 PM

Spring 2019 Name: SOLUTION

Dr. Havlicek Student Num:__________

**Directions:** This is an open notes test. You may use a clean copy of the course notes as published on the course web site. Other materials are not allowed. You have 120 minutes to complete the test. All work must be your own.

SHOW ALL OF YOUR WORK for maximum partial credit!

## GOOD LUCK!

SCORE:

1. (20) ________

2. (20) ________

3. (20) ________

4. (20) ________

5. (20) ________

TOTAL (100):

*On my honor, I affirm that I have neither given nor received inappropriate aid in the completion of this test.*

Name:__________ Date:__________

# POSTING OF FINAL GRADES

It is the policy of the University of Oklahoma that grades will not be posted by student name, social security number, or student number.

Therefore, final grades for this class will be posted using CODE NAMES, but only AT YOUR REQUEST.

Indicate your preference by marking ONE of the boxes below. If you request for your grade to be posted, you MUST provide a CODE NAME.

☐ Do NOT post my final grade.

☐ DO post my final grade.

Code Name: ______________________________

1. **20 pts.** True or False. Mark *True* only if the statement is **always** true.

TRUE FALSE

✓ ___ (a) **3 pts.** If two images are discrete and periodic with the same period, then their wraparound convolution equals their linear convolution.

✓ ___ (b) **3 pts.** For convolving two $N \times N$ digital images, multiplying DFT's instead of directly implementing the convolution reduces the computational complexity from $N^4$ to $N^2 \log N^2$.

✓ ___ (c) **3 pts.** Homomorphic filtering can be used to transform a multiplicative noise problem into an additive noise problem.

✓ ___ (d) **3 pts.** Edge thinning and edge linking are not required after application of the basic LoG edge detector (without thresholding).

✓ ___ (e) **3 pts.** The trimmed mean filter $TM_{P/2}$ is robust (nearly optimal) for removing both Laplacian and Gaussian additive noise.

✓ ___ (f) **3 pts.** The basic idea behind DPCM is that pixel differencing tends to reduce the entropy of typical images like *lena* or *cameraman*.

OH MY! (g) **2 pts.** The Wiener filter was first introduced in 1945 by famous MIT electrical engineer Oscar F. Mayer.

2. **20 pts.** Short Answer.

(a) **4 pts.** Name an effective technique for lossless entropy reduction.

DPCM (Notes p. 7-20)

(b) **4 pts.** Explain what people mean when they describe the median filter as a "low-pass" filter. Strictly speaking, is this correct? What is the frequency response of the median filter?

- Median filter tends to remove small objects which are generally rich in high frequencies.
- Not correct because not all high frequencies are removed... the filter is nonlinear. IT DOES NOT HAVE A FREQUENCY RESPONSE.

(c) **4 pts.** Can the gray scale morphological opening be implemented by multiplication of DFT's? Briefly explain.

NO. OPEN is a nonlinear filter. It does not have a frequency response.

(d) **4 pts.** What is the main difference between the results that are obtained from applying the gray scale morphological filters OPEN and CLOSE?

Both smooth noise and preserve edges.

- CLOSE removes negative spikes but not positive spikes.
- OPEN removes positive spikes but not negative spikes.

(e) **4 pts.** Can the gray scale morphological filters ERODE and DILATE be described as Order Statistic Filters? Briefly explain/show.

YES: $\text{ERODE}[I, B] = \text{MIN}[I, B]$

$\text{DILATE}[I, B] = \text{MAX}[I, B]$

3. **20 pts.** The $4 \times 4$ image $\mathbf{I}$ shown on the left below has 8-bit pixels in the range $0 \leq I(m, n) \leq 255$. This image is transmitted through a communication channel where it is corrupted by IID additive white Laplacian noise. The received image $\mathbf{J}$ is shown on the right below.

$\mathbf{I}$ =

| | | | |
|---|---|---|---|
| 32 | 32 | 32 | 32 |
| 120 | 120 | 32 | 32 |
| 120 | 120 | 120 | 32 |
| 120 | 120 | 120 | 120 |

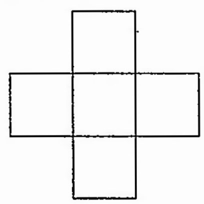
$\mathbf{J}$ =

| | | | |
|---|---|---|---|
| 30 | 32 | 28 | 35 |
| 125 | 134 | 39 | 36 |
| 117 | 115 | 107 | 27 |
| 123 | 120 | 122 | 119 |

(a) **15 pts.** Choose the best OS or morphological filter (MED, AVE, INNER_AVE, ERODE, DILATE, OPEN, CLOSE, *etc.*) to denoise the received image by attenuating the transmission noise. Use the structuring element/window $\mathbf{B}$ = CROSS(5) shown below. **Show** the denoised image $\mathbf{K}$ in the space provided at the bottom of this page. Handle edge effects by replication.

MEDIAN is best for Laplacian noise

$\mathbf{B}$ =



**Note:** Work space is provided on the following page.

(b) **5 pts.** Compute the ISNR for the denoised image $\mathbf{K}$.

**Hint:** $\text{ISNR}(\mathbf{K}) = 10 \log_{10} \dfrac{\text{MSE}(\mathbf{J})}{\text{MSE}(\mathbf{K})}$

Show the denoised image here:

$\mathbf{K}$ =

| | | | |
|---|---|---|---|
| 30 | 32 | 32 | 35 |
| 125 | 115 | 39 | 36 |
| 117 | 117 | 107 | 36 |
| 123 | 120 | 120 | 119 |

ISNR = 2.1310 dB

Work Space for Problem 2...

$$MSE(I) = \frac{\sum (I-J)^2}{16} = \frac{557}{16} = 34.8125$$

$$MSE(K) = \frac{\sum (I-K)^2}{16} = \frac{341}{16} = 21.3125$$

$$ISNR = 10\log_{10}\frac{MSE(J)}{MSE(K)} = 10\log_{10} 1.63343 \approx 2.1310 \text{ dB}$$

$$\text{Extended } J \quad \begin{array}{c|cccc|c} & 30 & 32 & 28 & 35 & \\ \hline 30 & 30 & 32 & 28 & 35 & 35 \\ 125 & 125 & 134 & 39 & 36 & 36 \\ 117 & 117 & 115 & 107 & 27 & 27 \\ 123 & 123 & 120 & 122 & 119 & 119 \\ \hline & 123 & 120 & 122 & 119 & \end{array} \qquad K = \begin{bmatrix} 30 & 32 & 32 & 35 \\ 125 & 115 & 39 & 36 \\ 117 & 117 & 107 & 36 \\ 123 & 120 & 120 & 119 \end{bmatrix}$$

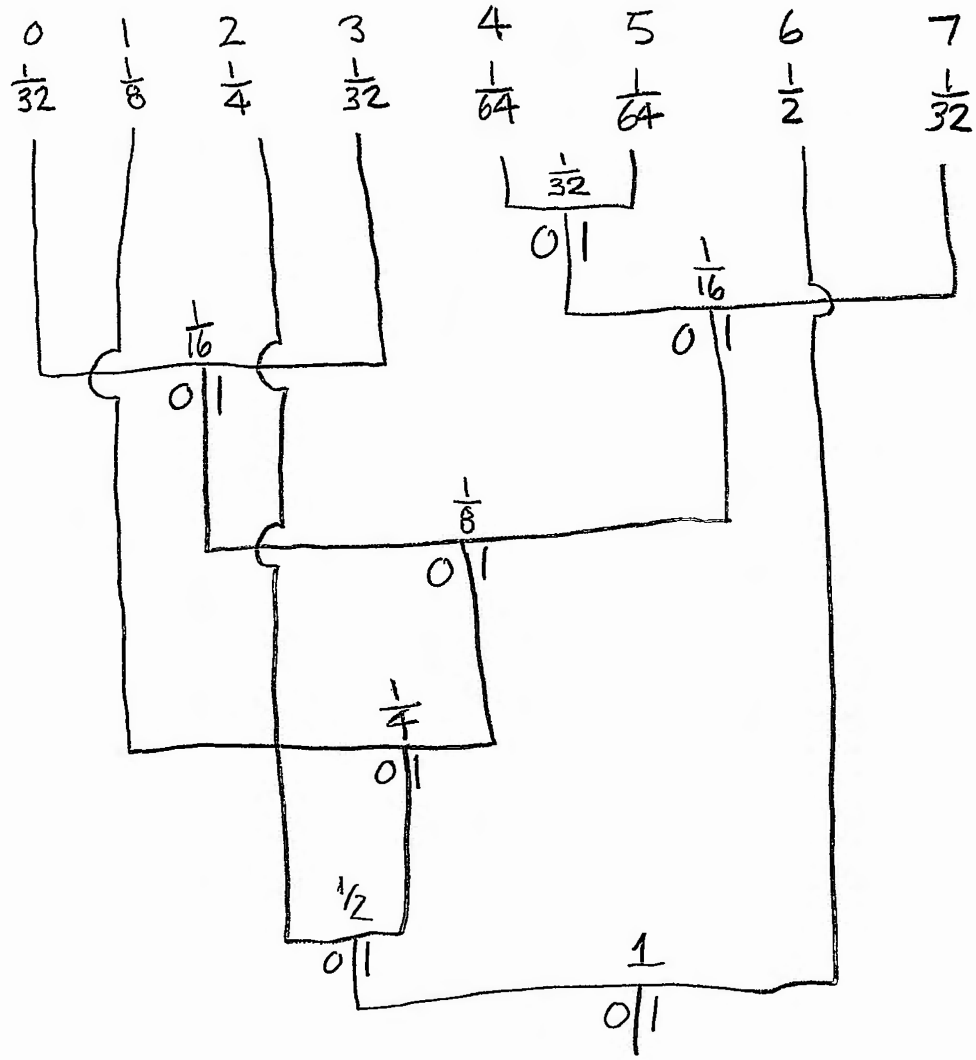
$$I-J = \begin{bmatrix} 2 & 0 & 4 & -3 \\ -5 & -14 & -7 & -4 \\ 3 & 5 & 13 & 5 \\ -3 & 0 & -2 & 1 \end{bmatrix} \qquad I-K = \begin{bmatrix} 2 & 0 & 0 & -3 \\ -5 & 5 & -7 & -4 \\ 3 & 3 & 13 & -4 \\ -3 & 0 & 0 & 1 \end{bmatrix}$$

$$(I-J)^2 = \begin{bmatrix} 4 & 0 & 16 & 9 \\ 25 & 196 & 49 & 16 \\ 9 & 25 & 169 & 25 \\ 9 & 0 & 4 & 1 \end{bmatrix} \qquad (I-K)^2 = \begin{bmatrix} 4 & 0 & 0 & 9 \\ 25 & 25 & 49 & 16 \\ 9 & 9 & 169 & 16 \\ 9 & 0 & 0 & 1 \end{bmatrix}$$

4. **20 pts.** Gray scale digital images **I** with 3 bits per pixel and gray levels in the range $\{0, 1, \ldots, 7\}$ are modeled as coming from an information source with the following source symbol probabilities (normalized histogram):

| $k$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $p_{\mathbf{I}}(k)$ | $\frac{1}{32}$ | $\frac{1}{8}$ | $\frac{1}{4}$ | $\frac{1}{32}$ | $\frac{1}{64}$ | $\frac{1}{64}$ | $\frac{1}{2}$ | $\frac{1}{32}$ |

(a) **12 pts.** Design a Huffman code to encode these images.




| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| C(k) | 01100 | 010 | 00 | 01101 | 011100 | 011101 | 1 | 01111 |
| L[C(k)] | 5 | 3 | 2 | 5 | 6 | 6 | 1 | 5 |

Problem 4 cont...

(b) **4 pts.** Find the expected BPP (bits per pixel) and CR (compression ratio) for the coded images $C(\mathbf{I})$.

$$BPP(I) = 3$$

$$BPP(\hat{I}) = \sum_{k=0}^{7} p_I(k) L[c(k)]$$

$$= (\tfrac{1}{32})5 + (\tfrac{1}{8})3 + (\tfrac{1}{4})2 + (\tfrac{1}{32})5 + (\tfrac{1}{64})6 + (\tfrac{1}{64})6 + (\tfrac{1}{2})1 + (\tfrac{1}{32})5$$

$$= \tfrac{5}{32} + \tfrac{3}{8} + \tfrac{2}{4} + \tfrac{5}{32} + \tfrac{6}{64} + \tfrac{6}{64} + \tfrac{1}{2} + \tfrac{5}{32}$$

$$= \frac{5+12+16+5+3+3+16+5}{32} = \frac{65}{32} = \underline{\underline{2.03125}}$$

$$CR = \frac{BPP(I)}{BPP(\hat{I})} = \frac{3}{2.03125} = \underline{\underline{1.47692}}$$

(c) **4 pts.** Does your code achieve the theoretical lower bound on BPP for the coded images? Explain why or why not.

<u>YES</u>. Since all the symbol probabilities are integer powers of 2, this code achieves the theoretical lower bound on BPP.

5. **20 pts.** Consider the original $512 \times 512$ grayscale *Barbara* image $\mathbf{I}$ shown below:

Each pixel has 8 bits. During transmission, the image is corrupted by salt-and-pepper noise and additive white Gaussian noise with standard deviation $\sigma = 12$. The received image $\mathbf{J} = \mathbf{I} + \mathbf{N}$ is shown below:

**Problem 5, cont...**

The following seven filters are applied to **J** in an effort to restore the image by denoising:

(a) $\mathbf{K} = \text{AVE}[\mathbf{J}, \mathbf{B}]$, with $\mathbf{B} = \text{SQUARE}(25)$.

(b) $\mathbf{K} = \text{MED}[\mathbf{J}, \mathbf{B}]$, with $\mathbf{B} = \text{SQUARE}(25)$.

(c) $\mathbf{K} = \text{CLOSE}[\mathbf{J}, \mathbf{B}]$, with $\mathbf{B} = \text{SQUARE}(25)$.

(d) $\mathbf{K} = \text{OPEN}[\mathbf{J}, \mathbf{B}]$, with $\mathbf{B} = \text{SQUARE}(25)$.

(e) $\mathbf{K} = \text{CLOSE-OPEN}[\mathbf{J}, \mathbf{B}]$, with $\mathbf{B} = \text{SQUARE}(25)$.

(f) A $128 \times 128$ linear Gaussian low-pass filter with space constant $\sigma = 3$, similar to Example 3 on page 5.61 of the course notes.

(g) A $256 \times 256$ linear Gaussian high-pass filter with space constant $\sigma = 48$, similar to Example 6 on page 5.79 of the course notes.

Filters (a)-(e) are implemented directly in the space domain with edge effects handled by replication. Filters (f) and (g) are implemented by pointwise multiplication of DFT's with appropriate zero padding to obtain linear convolution.

Label the seven filtered images shown below on pages 7-9.

$\boxed{\text{CLOSE-OPEN}}$

- Lots of smoothing but edges are sharp.
  → Can't be average or Gaussian low-pass.

- Image is grossly oversmoothed.
  → This results from applying four 5x5 filtering operations: two dilations and two erosions.
  → Must be CLOSE-OPEN.

Problem 5, cont...

GAUSSIAN LOW-PASS

- Clearly linear low-pass.
  → Must be AVE or Gaussian
- Dark bands around edges result from zero padding
  → edge effects <u>not</u> handled by replication
  → Can't be AVE
  → Must be Gaussian low-pass

MEDIAN

- removes both Gaussian noise and salt-and-pepper noise.
- Edges not blurred

→ Must be median

AVE

- There is some blurring, but it is very mild
- Effectively removes Gaussian noise
- Some artifacts from salt-and-pepper noise remain...
  salt-and-pepper <u>not</u> removed effectively

→ Must be AVE

Problem 5, cont...

OPEN

- Because EROSION is done first, the salt is removed
- But the pepper remains.

→ Must be OPEN

GAUSSIAN HIGH PASS

- Obviously a high-pass filter
- Noise is amplified instead of attenuated.

→ Must be Gaussian High-Pass.

CLOSE

- DILATION is done first. So pepper is removed.
- But salt remains

→ Must be CLOSE.