# ECE 5273 <br> Test 2 

Friday, May 14, 2021
10:30 AM - 12:30 PM
Spring 2021
Name: SOLUTION
Dr. Havlicek
Student Num:

Directions: This is an open notes test. You may use a clean copy of the course notes as published on the course web site and a calculator. Other materials are not allowed. You have 120 minutes to complete the test. All work must be your own. You may work the test on this test paper or you may use your own blank paper. Upload a scan or photograph of your test paper to the course Canvas page no later than 12:45 PM on Friday, May 14, 2021.

SHOW ALL OF YOUR WORK for maximum partial credit!

## GOOD LUCK!

SCORE:

1. (20) $\qquad$
2. (20) $\qquad$
3. (20) $\qquad$
4. (20) $\qquad$
5. (20) $\qquad$

TOTAL (100):

On my honor, I affirm that I have neither given nor received inapproprate aid in the completion of this test.
Name:
Date:

1. 20 pts. True or False. Mark True only if the statement is always true.

## TRUE FALSE

$x$
(a) 3 pts. DOG filters are very useful in human visual modling. Notes p. 5.98
(b) 3 pts. Any grayscale morphological filter can be implemented by pointwise multiplication of DFT's if sufficient zero padding is applied. Morphological filters are nonlinear
(c) 3 pts. Linear filtering can be implemented by pointwise multiplication of DFT"s if sufficient zero padding is apphed. Notes P. 5.33
(d) 3 pts. In grayscale template matching, the normalized cross-correlation $\hat{C}_{\mathbf{B}_{\mathbf{T}^{\circ}}((2, j), \mathbf{T}}$ always takes a value between 0 and 1 (inclusive). Notes p. 8.4-2
(e) 3 pts. One of the mam features of Canny's Edge Detector is that it computes a second derivative in the direction normal to the edge. Notes p. 8.87
(f) 3 pts. In anisotropic diffusion, the diffusion coefficients control the inhibition of smoothing over edges and encourage smoothing in non-edge image regions. Notes p. 8. 94
(g) 2 pts. The Salesman used to be an ECE professor at OU, but he got thrown out for saying that Trump incited the Capitol insurrection.
2. 20 pts. The ECE department photocopier has developed a problem. There are vertical black lines on all of the copies. The lines are always one pixel wide and are always separated by at least 64 columns. Because it is a hardware problem, these same vertical black lines show up when the photocopier is used to make scans as well. Two examples are given below:





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Unfortunately, it will be at least three weeks before a technician can come out to repair the problem. In the meantime, here is the plan: instead of copying, everybody will use the photocopier to scan instead. A nonlinear digital gray scale filter will then be applied to remove the vertical black lines from the scanned images, which can then be printed.

Because of your excellent performance in ECE 5273, you have been asked to design the filter. Carefully explain which filtering operation you recommend, which window or structuring element, and the reasons for your choices.

Work Space for Problem 2...

- The window should be as small as possible to avoid making undesirable changes to good pixels.
- Since the bad lines are alwaves vertical, always one pixel wide, and always separated by at least 64 columans, the window should be a ROW.
$\rightarrow$ Row ls) is the smallest window that will weak.
- As can be seen in the example images, the bad black lines will generally cross both dark and light structure in the image. MEDIAN will replace the bad pixels with a value that is similar to the majority of the pixels on the neighborhood and it will not blur edges.
$\Rightarrow$ USE MEDIAN with windows $B=$ ROW (3)

CLOSE would also work, but it is not as good as MEDIAN because it involves two primitive filtering operations... so the effective spatial extent is larger [no wis)] and it would make more changes to the good pixels.
3. 20 pts. The $5 \times 5$ image I shown below has 3-bit pixels in the range $0 \leq I(m, n) \leq 7$ :

$I=$| 4 | 7 | 2 | 5 | 3 |
| :--- | :--- | :--- | :--- | :--- |
| 1 | 5 | 4 | 7 | 4 |
| 0 | 5 | 7 | 2 | 3 |
| 3 | 7 | 1 | 2 | 4 |
| 2 | 4 | 4 | 4 | 5 |

A grayscale morphological EROSION is to be performed using the structuring element $\mathbf{B}=\operatorname{CROSS}(5)$ shown here:


Implement the EROSION filter as a stack filter to compute the filtered output value for the middle pixel $\mathbf{I}(2,2)$ shown above as a large boldface " 7. "

Note: you do not have to apply the filter at any other pixels! Just apply it at the middle pixel $\mathbf{I}(2,2)$ only to find the filtered output value for that pixel only.

Hint: extract the window set $\mathrm{B} \circ \mathrm{I}(2,2)$ and arrange it as a horizontal 1 D array. Then compute the binary threshold decomposition for the window set and apply a binary EROSION filteı to cach binary the cshold signal. Finally, combine the results using the stacking property to get the final gray scale value for the filtered pixel.

More workspace is provided on the following page in case you need it.

4. 20 pts . Pixels in the $6 \times 6$ image I shown below take values in the range $\{0,1,2, \ldots, 99\}$. The image is sent through a communication channel where it is corrupted by additive noise. The received image $\mathbf{J}$ is also shown below.

$I=$| $\mathbf{1 1}$ | $\mathbf{1 1}$ | $\mathbf{1 2}$ | $\mathbf{1 2}$ | $\mathbf{1 3}$ | 13 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1 1}$ | $\mathbf{0}$ | $\mathbf{1 2}$ | $\mathbf{1 3}$ | $\mathbf{1 3}$ | 13 |
| $\mathbf{1 2}$ | $\mathbf{1 2}$ | $\mathbf{1 3}$ | $\mathbf{1 3}$ | $\mathbf{1 3}$ | $\mathbf{1 4}$ |
| $\mathbf{1 2}$ | $\mathbf{1 3}$ | $\mathbf{1 3}$ | $\mathbf{1 3}$ | $\mathbf{1 4}$ | $\mathbf{1 4}$ |
| $\mathbf{1 3}$ | $\mathbf{1 3}$ | 13 | $\mathbf{1 4}$ | $\mathbf{0}$ | $\mathbf{1 4}$ |
| $\mathbf{1 3}$ | 13 | 13 | $\mathbf{1 4}$ | $\mathbf{1 4}$ | 14 |



Design a nonlinear filter to restore the received image by attenuating the noise. Handle edge effects by replication. Explain your solution. Show the restored image $\mathbf{K}$ below and compute the ISNR. There is workspace on the following page.
The noise mtrurduces positive spiked that need to be removed. But the image contains negative spikes that need to be retained. so we need to use an asymmetric souther that attenuates positive spikes only. To minimize distortion, ne Meed to use small 21 wind lad.

$$
\Rightarrow \text { USE OPEN with } B=\operatorname{CROSS}(5)
$$

Show the restored mage here.

$K=$| 11 | 11 | 12 | 12 | 13 | 13 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 11 | 0 | 12 | 13 | 13 | 13 |
| 12 | 12 | 13 | 13 | 13 | 14 |
| 12 | 13 | 13 | 13 | 14 | 14 |
| 13 | 13 | 13 | 14 | 0 | 14 |
| 13 | 13 | 14 | 14 | 14 | 14 |

$$
\mathrm{ISNR}=45.68 \mathrm{~dB}
$$

More Workspace for Problem ${ }^{4} 19$.

$$
\begin{aligned}
& \begin{array}{cccccccccc} 
& 11 & 11 & 0 & 11 & 12 & 12 & 13 & 13 \\
0 & 0 & 0 & 0 & 12 & 13 & 13 & 13 \\
\text { ERODE } & 11 & 11 & 0 & 12 & 13 & 13 & 13 & 13 \\
12 & 12 & 12 & 13 & 13 & 0 & 14 & 14
\end{array} \\
& 121213130000 \\
& \begin{array}{llllllll}
13 & 13 & 13 & 13 & 14 & 0 & 14 & 14 \\
13 & 13 & 13 & 14 & 0 & 14
\end{array} \\
& |K-I|
\end{aligned}
$$

$$
\begin{aligned}
& \operatorname{MSE}(k)=\frac{\sum_{i=0}^{5} \sum_{j=0}^{5}[k(i, j)-I(i, j)]^{2}}{36} \\
& \operatorname{MSE}(J)=\frac{\sum_{i=0}^{5} \sum_{j=0}^{5}[J(i, j)-I(i, j)]^{2}}{36} \\
& =\frac{1}{36} \\
& =\frac{5 \cdot(86)^{2}}{36}=\frac{36,980}{36} \\
& I S N R=-10 \log _{10} \frac{M S E(J)}{M S E(K)}=10 \log _{10} \frac{36,980}{1}=45.67967 \mathrm{~dB}
\end{aligned}
$$

5. 20 pts. Gray scale digital images I with 3 bits per pixel and gray levels in the range $\{0,1, \ldots, 7\}$ are modeled as coming from an information source with the following source symbol probabilities (normalized histogram):

| $k$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $p_{\mathbf{I}}(k)$ | $\frac{1}{32}$ | $\frac{1}{32}$ | $\frac{1}{8}$ | $\frac{1}{16}$ | $\frac{1}{2}$ | $\frac{1}{16}$ | $\frac{1}{16}$ | $\frac{1}{8}$ |

(a) 12 pts. Design a Huffman code to encode these images.


Problem 5 cont. .
(b) 4 pts. Find the expected BPP (bits per pixel) and CR (compression ratio) for the coded images $C(\mathrm{I})$.
Original $B P P=3$

$$
\begin{aligned}
& \text { Original } B P P=3 \\
& \begin{aligned}
\text { Coded } B P P & = \\
& \quad 0 \cdot \frac{1}{32}+5 \cdot \frac{1}{32}+3 \cdot \frac{1}{6}+4 \cdot \frac{1}{16}+1 \cdot \frac{1}{2}+4 \cdot \frac{1}{16}+4 \cdot \frac{1}{16}+3 \cdot \frac{1}{8} \\
& =\frac{5}{32}+\frac{5}{32}+\frac{3}{8}+\frac{4}{16}+\frac{1}{2}+\frac{4}{16}+\frac{4}{16}+\frac{3}{8} \\
& =\frac{5+5+12+8+16+8+8+12}{32}=\frac{74}{32}=2.3125
\end{aligned}
\end{aligned}
$$

$$
C R=\frac{3}{2.3125}=1.2973
$$

(c) 4 pts. Does your code achieve the theoretical lower bound on BPP for the coded images? Explain why or why not.

YES - because the source symbol probubillties are all integer powers of 2 .


