ECE 5273
Test 2

Friday, May 13, 2022
10:30 AM - 12:30 PM

Spring 2022 Name: SOLUTION

Dr. Havlicek Student Num: ____________

Directions: This is an open notes test. You may use a clean copy of the course notes as published on the course web site and a calculator. Other materials are not allowed. You have 120 minutes to complete the test. All work must be your own.

SHOW ALL OF YOUR WORK for maximum partial credit!

GOOD LUCK!

SCORE:

1. (20) ________

2. (20) ________

3. (20) ________

4. (20) ________

5. (20) ________

TOTAL (100):

____

On my honor, I affirm that I have neither given nor received inappropriate aid in the completion of this test.

Name: ________________ Date: ________________

1. **20 pts.** True or False. Mark *True* only if the statement is always true.

TRUE	FALSE	
X		(a) **3 pts.** If two digital images are both periodic with the same period, then their wraparound convolution equals their linear convolution. Notes p. 5.21
X		(b) **3 pts.** A linear translation invariant (LTI) digital image processing system can be completely characterized by the system impulse response. Notes p. 5.87
	X	(c) **3 pts.** The fact that the median filter is so good at removing impulsive noise can be explained by observing that the filter frequency response is low pass. Nonlinear → No freq Resp Notes p. 6.19
X		(d) **3 pts.** An image with a flat histogram cannot be compressed by a variable wordlength code alone. Notes p. 7.18
X		(e) **3 pts.** The only lossy part of the baseline JPEG algorithm involves quantizing discrete cosine transform (DCT) coefficients computed on 8 × 8 blocks of pixels. Notes p. 7.73
X		(f) **3 pts.** One of the main features of Canny's Edge Detector is that it computes a second derivative in the direction normal to the edge. Notes p. 8.87
RIDICULOUS!		(g) **2 pts.** The DoG filter (difference-of-Gaussians) is mostly useful for removing unwanted cats from digital images.

2. **20 pts**. The 512×512 image **I** shown at left below has 8-bit pixels in the range $0 \leq I(m,n) \leq 255$. This image is transmitted through a communication channel where it is corrupted by IID additive white Gaussian noise with standard deviation $\sigma_N = 16$. The received image **J** is at right below.

I

J

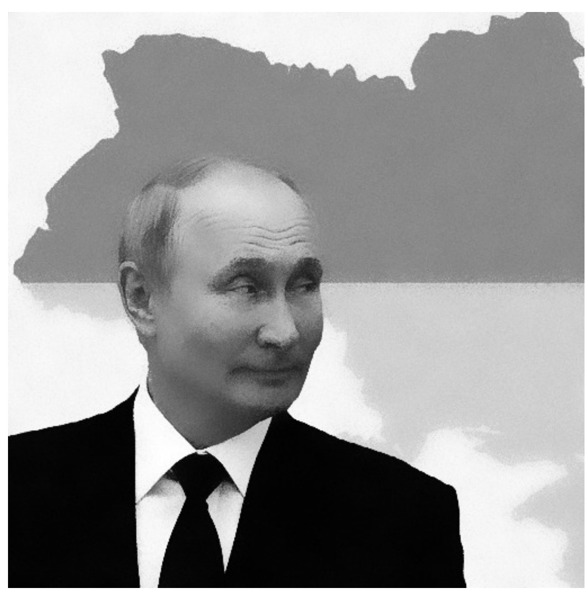
The following two filters are applied to perform denoising:

- A 512×512 Gaussian low-pass filter with space constant $\sigma = 2.50$, implemented as described on pages 5.64-5.65 of the course notes,
- A bilateral filter with Gaussian weighting functions having $\sigma_G = 8.0$ and $\sigma_H = 35.5$.

Identify and label the two filtered images shown below:

Gaussian

- Blurs Image
- Linear convolution leaves dark border



Bilateral Filter

- Sharp Edges
- No Blur
- Removes Noise Better

3. **20 pts.** Pixels in the 6×6 image **I** shown below take values in the range $\{0, 1, 2, \ldots, 99\}$. The image is sent through a communication channel where it is corrupted by additive noise. The received image **J** is also shown below.

I =

11	11	11	70	70	70
11	11	11	70	70	70
11	11	11	70	70	70
11	11	11	70	70	70
11	11	11	70	70	70
11	11	11	70	70	70

J =

11	11	11	70	70	70
11	4	11	70	70	70
11	11	11	18	5	70
11	11	11	21	11	70
11	11	11	13	70	70
11	11	11	70	70	70

Design a nonlinear filter to restore the received image by attenuating the noise. Handle edge effects by replication. Explain your solution. Show the restored image **K** below and compute the ISNR. There is workspace on the following two pages.

Because of the multiple noise hits on the high side of the edge, the following windows won't work: ROW(3), COL(3), CROSS(5), SQUARE(9).
⇒ Need to use COL(5) with MAX/DILATE to restore all pixels.

USE MAX/DILATE with B=COL(5)

Show the restored image here:

K =

11	11	11	70	70	70
11	11	11	70	70	70
11	11	11	70	70	70
11	11	11	70	70	70
11	11	11	70	70	70
11	11	11	70	70	70

ISNR= ∞

More Workspace for Problem 3...

K = DILATE(J,B) = MAX(J,B) = I ✓

11	11	11	70	70	70
11	11	11	70	70	70
11	11	11	70	70	70
11	11	11	70	70	70
11	11	11	70	70	70
11	11	11	70	70	70

$|I-J|$

0	0	0	0	0	0
0	7	0	0	0	0
0	0	0	52	65	0
0	0	0	49	59	0
0	0	0	57	0	0
0	0	0	0	0	0

$|I-J|^2$

0	0	0	0	0	0
0	49	0	0	0	0
0	0	0	2704	4225	0
0	0	0	2401	3481	0
0	0	0	3249	0	0
0	0	0	0	0	0

$$MSE(J) = \left[7^2 + 52^2 + 65^2 + 49^2 + 59^2 + 57^2\right] / 36$$

$$= \frac{1}{36}\left[49 + 2704 + 4225 + 2401 + 3481 + 3249\right] = \frac{16{,}109}{36} = 447.472$$

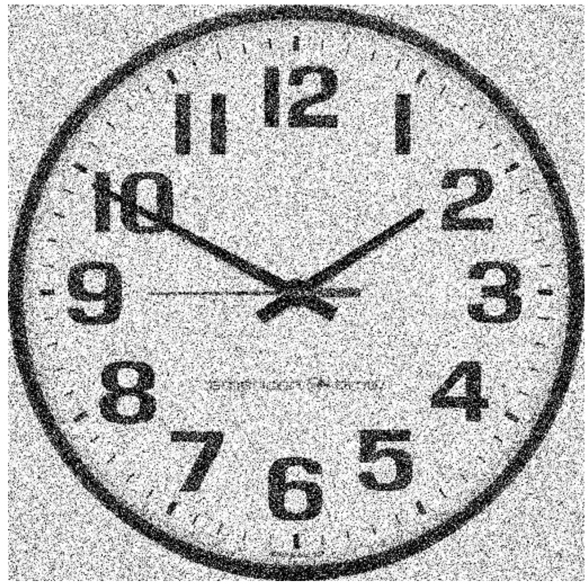
$$K = I \Rightarrow MSE(K) = 0$$

$$ISNR(K) = 10 \log_{10} \frac{MSE(J)}{MSE(K)} = \lim_{\varepsilon \to 0} 10 \log_{10} \frac{447.472}{\varepsilon} \longrightarrow \infty \;///$$

4. **20 pts**. The 512×512 image **I** shown at left below has 8-bit pixels in the range $0 \leq I(m,n) \leq 255$. This image is transmitted through a communication channel where it is corrupted by IID additive white Gaussian noise with standard deviation $\sigma_N = 100$. The received image **J** is at right below.

I

J

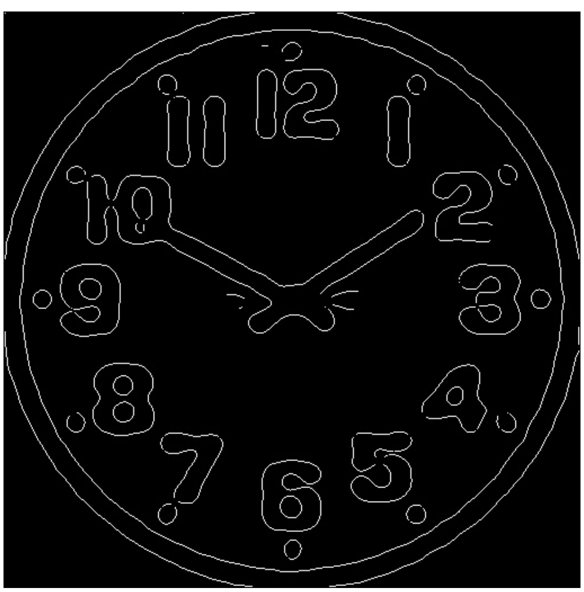


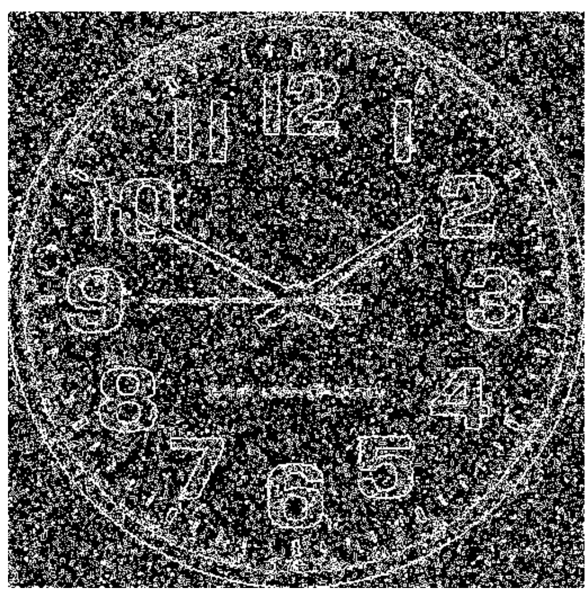
Two edge detectors are applied:

- Sobel, where the threshold shown on page 8.54 of the notes is set to 42.
- LoG with a space constant of $\sigma = 7.00$. ZC thresholding is applied with a threshold of 0.10.

Identify and label the two resulting edge maps shown below:

LoG





Sobel

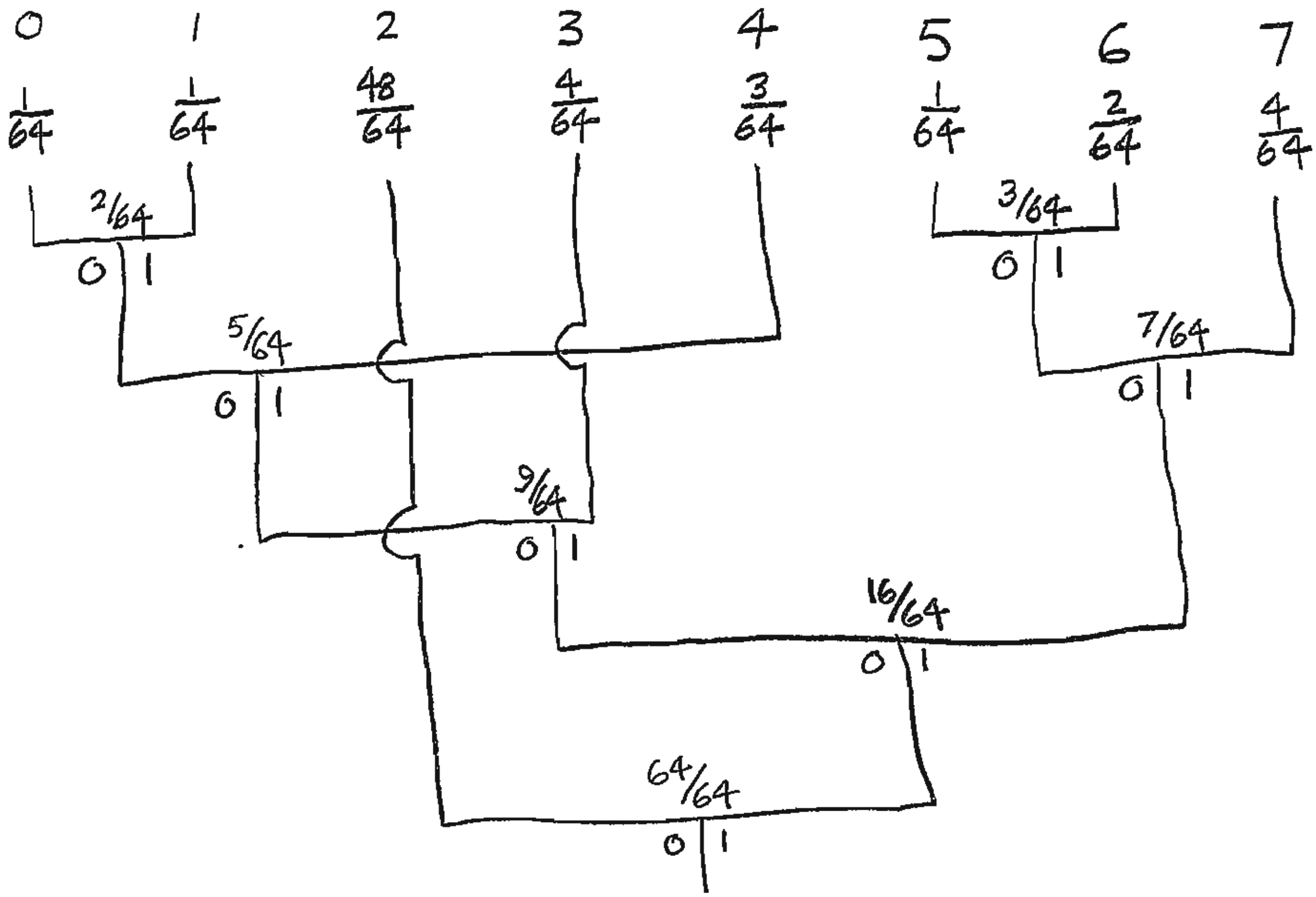
- Eliminates noise
- Rounds corners
- Detects only medium & large scale edges

- Detects all edges
- High sensitivity to noise

5. **20 pts.** Gray scale digital images **I** with 3 bits per pixel and gray levels in the range $\{0, 1, \ldots, 7\}$ are modeled as coming from an information source with the following source symbol probabilities (normalized histogram):

k	0	1	2	3	4	5	6	7
$p_{\mathbf{I}}(k)$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{3}{4}$	$\frac{1}{16}$	$\frac{3}{64}$	$\frac{1}{64}$	$\frac{1}{32}$	$\frac{1}{16}$

(a) **12 pts.** Design a Huffman code to encode these images.



k	0	1	2	3	4	5	6	7
C(k)	10000	10001	0	101	1001	1100	1101	111
L(k)	5	5	1	3	4	4	4	3

Problem 5 cont...

(b) **4 pts.** Find the expected BPP (bits per pixel) and CR (compression ratio) for the coded images $C(\mathbf{I})$.

Original BPP = 3

$$\text{Coded BPP} = \overset{0}{5}\cdot\frac{1}{64} + \overset{1}{5}\cdot\frac{1}{64} + \overset{2}{1}\cdot\frac{48}{64} + \overset{3}{3}\cdot\frac{4}{64} + \overset{4}{4}\cdot\frac{3}{64} + \overset{5}{4}\cdot\frac{1}{64} + \overset{6}{4}\cdot\frac{2}{64} + \overset{7}{3}\cdot\frac{4}{64}$$

$$= \frac{1}{64}\left[5+5+48+12+12+4+8+12\right]$$

$$= \frac{106}{64} \doteq \underline{\underline{1.65625}}$$

$$CR = \frac{3}{1.65625} \approx \underline{\underline{1.8113}}$$

(c) **4 pts.** Does your code achieve the theoretical lower bound on BPP for the coded images? Explain why or why not.

<u>NO</u>. The symbol probabilities are <u>not</u> all integer powers of two.

$P(4) = \frac{3}{64}$, which is <u>not</u> a power of 2.