# ECE 5273
# Test 2

Friday, May 12, 2023
10:30 AM - 12:30 PM

Spring 2023 — Name: SOLUTION

Dr. Havlicek — Student Num: ____

**Directions:** This is an open notes test. You may use a clean copy of the course notes as published on the course web site and a calculator. Other materials are not allowed. You have 120 minutes to complete the test. All work must be your own.

SHOW ALL OF YOUR WORK for maximum partial credit!

## GOOD LUCK!

SCORE:

1. (20) ____
2. (20) ____
3. (20) ____
4. (20) ____
5. (20) ____

TOTAL (100):

*On my honor, I affirm that I have neither given nor received inappropriate aid in the completion of this test.*

Name: ____ Date: ____

1. **20 pts.** True or False. Mark *True* only if the statement is **always** true.

| TRUE | FALSE | |
|---|---|---|
| X | | (a) **3 pts.** If $I_1$ and $I_2$ are two 256 × 256 digital images, then their linear convolution $J = I_1 * I_2$ has a size of 511 × 511. Notes p. 5.44 |
| X | | (b) **3 pts.** For the idealized linear image deblurring problem, deconvolution is done using the inverse filter of the distortion. Notes p. 5.120 |
| X | | (c) **3 pts.** White noise is uncorrelated noise. Notes p. 6.3 |
| X | | (d) **3 pts.** Laplacian noise is an example of a heavy-tailed noise. Notes p. 6.9 |
| | X | (e) **3 pts.** The median filter is best among all order statistic filters at reducing Gaussian noise variance. Notes p. 6.67 |
| | X | (f) **3 pts.** One of the main advantages of the gradient-based edge detectors is that they are insensitive to noise. Notes p. 8.65 |
| OH | MY! | (g) **2 pts.** The DeSantis filter is a widely used nonlinear operator for removing Disney characters from any digital image. |

2. **20 pts.** Short Answer.

(a) **12 pts.** For the window (structuring element) CROSS(5), give the order statistic (OS) filter weights (coefficients) **A** for

i) **3 pts.** A median filter:

**A** = $[0\ 0\ 1\ 0\ 0]^T$

ii) **3 pts.** An average filter:

**A** = $[\frac{1}{5}\ \frac{1}{5}\ \frac{1}{5}\ \frac{1}{5}\ \frac{1}{5}]^T$

iii) **3 pts.** An OS filter to perform morphological dilation:

**A** = $[0\ 0\ 0\ 0\ 1]^T$

iv) **3 pts.** An OS filter to perform morphological erosion:

**A** = $[1\ 0\ 0\ 0\ 0]^T$

(b) **8 pts.** *Briefly* describe the **main** difference between the Laplacian of Gaussian (LoG) edge detector and the Canny edge detector.

Both are based on applying a low-pass Gaussian filter and then finding the zero crossings of a second derivative.

The main difference is which second derivative operator is used.

→ For the LoG, it is an approximation of the Laplacian $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$.

→ For the Canny, it is an approximation of the second derivative $\frac{\partial^2}{\partial \vec{n}^2}$ taken in a direction $\vec{n}$ that is normal to the edge.

3. **20 pts**. Consider the *Cameraman* image $\mathbf{I}$ shown below.

The size of the image is $256 \times 256$ pixels and each pixel has eight bits. Five grayscale morphological filters are applied, all with respect to the structuring element $\mathbf{B}$ = SQUARE(9), to define five new filtered images according to

$$\begin{aligned}
\mathbf{J}_M &= \text{MED}(\mathbf{I}, \mathbf{B}), \\
\mathbf{J}_E &= \text{ERODE}(\mathbf{I}, \mathbf{B}), \\
\mathbf{J}_D &= \text{DILATE}(\mathbf{I}, \mathbf{B}), \\
\mathbf{J}_O &= \text{OPEN}(\mathbf{I}, \mathbf{B}), \\
\mathbf{J}_C &= \text{CLOSE}(\mathbf{I}, \mathbf{B}).
\end{aligned}$$

Label the five output images shown on the next page.

$J_M$, Median

removes small objects, both bright and dark.

$J_O$ Open

small bright objects removed, small dark objects preserved.

$J_C$, Close

small dark objects removed, small bright objects preserved.

$J_D$ Dilate

the brightest.

$J_E$, Erode

the darkest.

4. **20 pts.** Pixels in the $6 \times 6$ image **I** shown below take values in the range $\{0, 1, 2, \ldots, 99\}$. The image is sent through a communication channel where it is corrupted by additive noise. The received image **J** is also shown below.

**I** =

| | | | | | |
|---|---|---|---|---|---|
| **11** | **11** | **12** | **12** | **13** | **13** |
| **11** | **12** | **12** | **13** | **13** | **13** |
| **12** | **12** | **13** | **13** | **13** | **14** |
| **12** | **13** | **13** | **13** | **14** | **14** |
| **13** | **13** | **13** | **14** | **14** | **14** |
| **13** | **13** | **13** | **14** | **14** | **14** |

**J** =

| | | | | | |
|---|---|---|---|---|---|
| **11** | **11** | **33** | **12** | **13** | **13** |
| **11** | **12** | **0** | **13** | **13** | **13** |
| **12** | **12** | **99** | **13** | **13** | **14** |
| **12** | **13** | **13** | **89** | **14** | **14** |
| **13** | **13** | **13** | **45** | **14** | **14** |
| **13** | **13** | **13** | **1** | **14** | **14** |

Design a nonlinear filter to restore the received image by attenuating the noise. Handle edge effects by replication. Explain your solution. Show the restored image **K** below and compute the ISNR. There is workspace on the following two pages.

The noise has ~~both~~ positive and negative spikes → USE MEDIAN.
The image has a diagonal structure. The BEST window is a 3-point antidiagonal  . That solution is shown first.

→ SQUARE(9) and ROW(3) are also good choices for the window. They are shown on subsequent pages and give close but slightly inferior MSE and ISNR.

Show the restored image here:

**K** =

| | | | | | |
|---|---|---|---|---|---|
| 11 | 11 | 12 | 12 | 13 | 13 |
| 11 | 12 | 12 | 13 | 13 | 13 |
| 12 | 12 | 13 | 13 | 13 | 14 |
| 12 | 13 | 13 | 13 | 14 | 14 |
| 13 | 13 | 13 | 14 | 14 | 14 |
| 13 | 13 | 13 | 13 | 14 | 14 |

ISNR= 41.7281 dB

More Workspace for Problem 4...

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|---|---|---|---|---|---|
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J with replication

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | 11 | 11 | 33 | 12 | 13 | 13 | 13 |
| 11 | 11 | 11 | 33 | 12 | 13 | 13 | 13 |
| 11 | 11 | 12 | 0 | 13 | 13 | 13 | 13 |
| 12 | 12 | 12 | 99 | 13 | 13 | 14 | 14 |
| 12 | 12 | 13 | 13 | 89 | 14 | 14 | 14 |
| 13 | 13 | 13 | 13 | 45 | 14 | 14 | 14 |
| 13 | 13 | 13 | 13 | 1 | 14 | 14 | 14 |
| 13 | 13 | 13 | 13 | 1 | 14 | 14 | |

|I-K|

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 |

K

MF w/ ADIAG(3)

| | | | | | |
|---|---|---|---|---|---|
| 11 | 11 | 12 | 12 | 13 | 13 |
| 11 | 12 | 12 | 13 | 13 | 13 |
| 12 | 12 | 13 | 13 | 13 | 14 |
| 12 | 13 | 13 | 13 | 14 | 14 |
| 13 | 13 | 13 | 14 | 14 | 14 |
| 13 | 13 | 13 | 13 | 14 | 14 |

$$MSE(K) = \frac{1}{36}\left[1^2\right] = \frac{1}{36}$$

$$MSE(J) = \frac{14{,}887}{36}$$ (see work on next page)

$$ISNR = 10\log_{10}\frac{MSE(J)}{MSE(K)} = 10\log_{10}\frac{14{,}887}{1} = 41.7281 \text{ dB}$$

More Workspace for Problem 4...

I

| | | | | | |
|---|---|---|---|---|---|
| 11 | 11 | 12 | 12 | 13 | 13 |
| 11 | 12 | 12 | 13 | 13 | 13 |
| 12 | 12 | 13 | 13 | 13 | 14 |
| 12 | 13 | 13 | 13 | 14 | 14 |
| 13 | 13 | 13 | 14 | 14 | 14 |
| 13 | 13 | 13 | 14 | 14 | 14 |

J

| | | | | | |
|---|---|---|---|---|---|
| 11 | 11 | 33 | 12 | 13 | 13 |
| 11 | 12 | 0 | 13 | 13 | 13 |
| 12 | 12 | 99 | 13 | 13 | 14 |
| 12 | 13 | 13 | 89 | 14 | 14 |
| 13 | 13 | 13 | 45 | 14 | 14 |
| 13 | 13 | 13 | 1 | 14 | 14 |

$|I-J|$

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 21 | 0 | 0 | 0 |
| 0 | 0 | 12 | 0 | 0 | 0 |
| 0 | 0 | 86 | 0 | 0 | 0 |
| 0 | 0 | 0 | 76 | 0 | 0 |
| 0 | 0 | 0 | 31 | 0 | 0 |
| 0 | 0 | 0 | 13 | 0 | 0 |

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$$MSE(J) = \frac{1}{36}\left[21^2 + 12^2 + 86^2 + 76^2 + 31^2 + 13^2\right]$$

$$= \frac{1}{36}\left[441 + 144 + 7{,}396 + 5{,}776 + 961 + 169\right]$$

$$= \frac{1}{36}\left[14{,}887\right]$$

$$\approx 413.528$$

4. **20 pts.** Pixels in the 6×6 image **I** shown below take values in the range $\{0, 1, 2, \ldots, 99\}$. The image is sent through a communication channel where it is corrupted by additive noise. The received image **J** is also shown below.

**I** =

| | | | | | |
|---|---|---|---|---|---|
| **11** | **11** | **12** | **12** | **13** | **13** |
| **11** | **12** | **12** | **13** | **13** | **13** |
| **12** | **12** | **13** | **13** | **13** | **14** |
| **12** | **13** | **13** | **13** | **14** | **14** |
| **13** | **13** | **13** | **14** | **14** | **14** |
| **13** | **13** | **13** | **14** | **14** | **14** |

**J** =

| | | | | | |
|---|---|---|---|---|---|
| **11** | **11** | **33** | **12** | **13** | **13** |
| **11** | **12** | **0** | **13** | **13** | **13** |
| **12** | **12** | **99** | **13** | **13** | **14** |
| **12** | **13** | **13** | **89** | **14** | **14** |
| **13** | **13** | **13** | **45** | **14** | **14** |
| **13** | **13** | **13** | **1** | **14** | **14** |

Design a nonlinear filter to restore the received image by attenuating the noise. Handle edge effects by replication. Explain your solution. Show the restored image **K** below and compute the ISNR. There is workspace on the following two pages.

Here is the solution for <u>MEDIAN</u> with <u>SQUARE(9)</u>.

– The calculation for MSE(J) is the same as on the previous solution.

Show the restored image here:

**K** =

| | | | | | |
|---|---|---|---|---|---|
| 11 | 11 | 12 | 13 | 13 | 13 |
| 11 | 12 | 12 | 13 | 13 | 13 |
| 12 | 12 | 13 | 13 | 13 | 14 |
| 12 | 13 | 13 | 14 | 14 | 14 |
| 13 | 13 | 13 | 14 | 14 | 14 |
| 13 | 13 | 13 | 13 | 14 | 14 |

ISNR= 36.9569 dB

More Workspace for Problem 4...

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J with replication

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 11 | 11 | 11 | 33 | 12 | 13 | 13 | 13 |
| 11 | 11 | 11 | 33 | 12 | 13 | 13 | 13 |
| 11 | 11 | 12 | 0 | 13 | 13 | 13 | 13 |
| 12 | 12 | 12 | 99 | 13 | 13 | 14 | 14 |
| 12 | 12 | 13 | 13 | 89 | 14 | 14 | 14 |
| 13 | 13 | 13 | 13 | 45 | 14 | 14 | 14 |
| 13 | 13 | 13 | 13 | 1 | 14 | 14 | 14 |
| 13 | 13 | 13 | 13 | 1 | 14 | 14 | 14 |

|I-K|

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 |

K MF w/ SQUARE(9)

| | | | | | |
|---|---|---|---|---|---|
| 11 | 11 | 12 | 13 | 13 | 13 |
| 11 | 12 | 12 | 13 | 13 | 13 |
| 12 | 12 | 13 | 13 | 13 | 14 |
| 12 | 13 | 13 | 14 | 14 | 14 |
| 13 | 13 | 13 | 14 | 14 | 14 |
| 13 | 13 | 13 | 13 | 14 | 14 |

$$MSE(K) = \frac{1}{36}\left[1^2 + 1^2 + 1^2\right] = \frac{3}{36}$$

$$MSE(J) = \frac{14{,}887}{36}$$ (same as in last solution)

$$ISNR = 10\log_{10}\frac{MSE(J)}{MSE(K)} = 10\log_{10}\frac{14{,}887}{3} = 36.9569\text{ dB}$$

4. **20 pts.** Pixels in the 6×6 image I shown below take values in the range $\{0, 1, 2, \ldots, 99\}$. The image is sent through a communication channel where it is corrupted by additive noise. The received image J is also shown below.

I =

| | | | | | |
|---|---|---|---|---|---|
| **11** | **11** | **12** | **12** | **13** | **13** |
| **11** | **12** | **12** | **13** | **13** | **13** |
| **12** | **12** | **13** | **13** | **13** | **14** |
| **12** | **13** | **13** | **13** | **14** | **14** |
| **13** | **13** | **13** | **14** | **14** | **14** |
| **13** | **13** | **13** | **14** | **14** | **14** |

J =

| | | | | | |
|---|---|---|---|---|---|
| **11** | **11** | **33** | **12** | **13** | **13** |
| **11** | **12** | **0** | **13** | **13** | **13** |
| **12** | **12** | **99** | **13** | **13** | **14** |
| **12** | **13** | **13** | **89** | **14** | **14** |
| **13** | **13** | **13** | **45** | **14** | **14** |
| **13** | **13** | **13** | **1** | **14** | **14** |

Design a nonlinear filter to restore the received image by attenuating the noise. Handle edge effects by replication. Explain your solution. Show the restored image K below and compute the ISNR. There is workspace on the following two pages.

Here is the solution for <u>MEDIAN</u> with <u>ROW(3)</u>.

- The calculation for MSE(J) is the same as in the previous two solutions.

Show the restored image here:

K =

| | | | | | |
|---|---|---|---|---|---|
| 11 | 11 | 12 | 13 | 13 | 13 |
| 11 | 11 | 12 | 13 | 13 | 13 |
| 12 | 12 | 13 | 13 | 13 | 14 |
| 12 | 13 | 13 | 14 | 14 | 14 |
| 13 | 13 | 13 | 14 | 14 | 14 |
| 13 | 13 | 13 | 13 | 14 | 14 |

ISNR= 35.7075 dB

More Workspace for Problem 4...

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J with replication

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 11 | 11 | 11 | 33 | 12 | 13 | 13 | 13 |
| 11 | 11 | 12 | 0 | 13 | 13 | 13 | 13 |
| 12 | 12 | 12 | 99 | 13 | 13 | 14 | 14 |
| 12 | 12 | 13 | 13 | 89 | 14 | 14 | 14 |
| 13 | 13 | 13 | 13 | 45 | 14 | 14 | 14 |
| 13 | 13 | 13 | 13 | 1 | 14 | 14 | 14 |

|I-K|

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 |

K MF w/ Row(3)

| | | | | | |
|---|---|---|---|---|---|
| 11 | 11 | 12 | 13 | 13 | 13 |
| 11 | 11 | 12 | 13 | 13 | 13 |
| 12 | 12 | 13 | 13 | 13 | 14 |
| 12 | 13 | 13 | 14 | 14 | 14 |
| 13 | 13 | 13 | 14 | 14 | 14 |
| 13 | 13 | 13 | 13 | 14 | 14 |

$$MSE(K) = \frac{1}{36}\left[1^2 + 1^2 + 1^2 + 1^2\right] = \frac{4}{36}$$

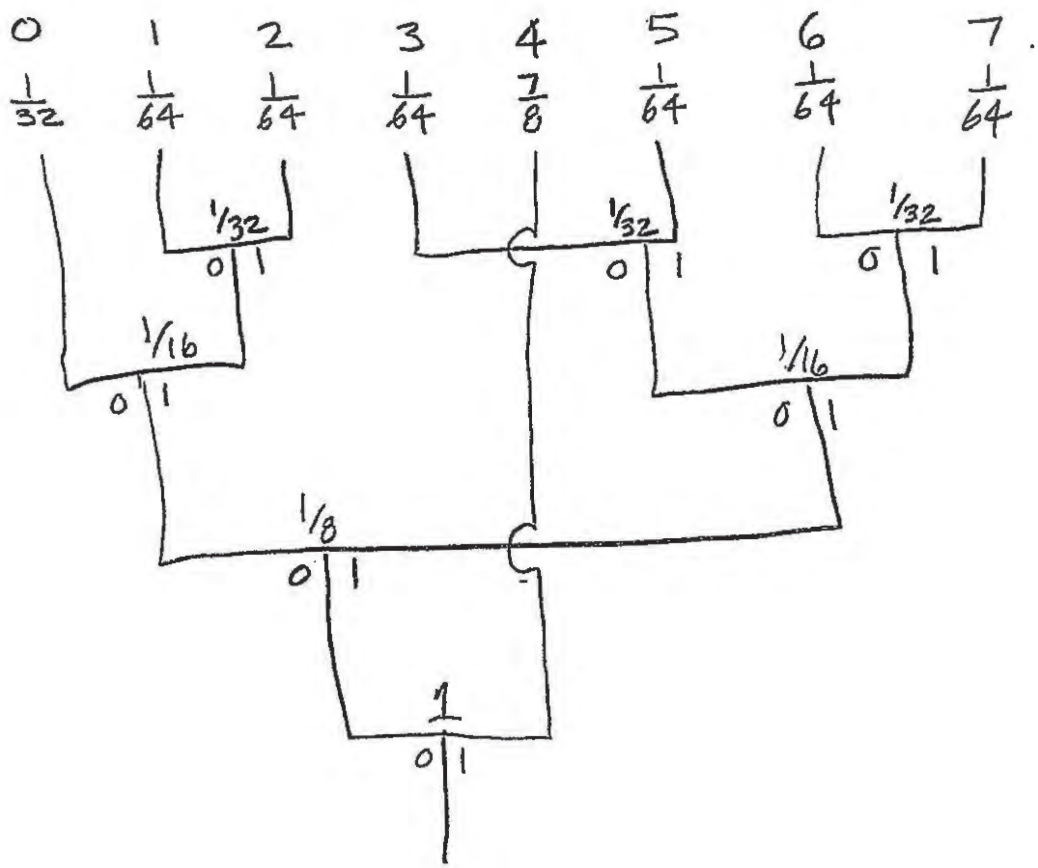
$$MSE(J) = \frac{14{,}887}{36}$$ (same as in the last two solutions)

$$ISNR = 10\log_{10}\frac{MSE(J)}{MSE(K)} = 10\log_{10}\frac{14{,}887}{4} = 35.7075 \text{ dB}$$

5. **20 pts.** Gray scale digital images **I** with 3 bits per pixel and gray levels in the range $\{0, 1, \ldots, 7\}$ are modeled as coming from an information source with the following source symbol probabilities (normalized histogram):

| $k$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $p_{\mathbf{I}}(k)$ | $\frac{1}{32}$ | $\frac{1}{64}$ | $\frac{1}{64}$ | $\frac{1}{64}$ | $\frac{7}{8}$ | $\frac{1}{64}$ | $\frac{1}{64}$ | $\frac{1}{64}$ |

(a) **12 pts.** Design a Huffman code to encode these images.




| K | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| C(K) | 000 | 0010 | 0011 | 0100 | 1 | 0101 | 0110 | 0111 |
| L(K) | 3 | 4 | 4 | 4 | 1 | 4 | 4 | 4 |

Problem 5 cont...

(b) **4 pts.** Find the expected BPP (bits per pixel) and CR (compression ratio) for the coded images $C(\mathrm{I})$.

$$\text{Coded BPP} = \overset{0}{3\cdot\tfrac{1}{32}} + \overset{1}{4\cdot\tfrac{1}{64}} + \overset{2}{4\cdot\tfrac{1}{64}} + \overset{3}{4\cdot\tfrac{1}{64}} + \overset{4}{1\cdot\tfrac{7}{8}} + \overset{5}{4\cdot\tfrac{1}{64}} + \overset{6}{4\cdot\tfrac{1}{64}} + \overset{7}{4\cdot\tfrac{1}{64}}$$

$$= \frac{3}{32} + \frac{24}{64} + \frac{7}{8} = \frac{6+24+56}{64} = \frac{86}{64} = \underline{1.34375}$$

$$CR = \frac{3}{86/64} = \frac{3\cdot 64}{86} = \frac{192}{86} = \underline{2.232558}$$

(c) **4 pts.** Does your code achieve the theoretical lower bound on BPP for the coded images? Explain why or why not.

**NO** $P(4) = \frac{7}{8}$ which is <u>not</u> an integer power of 2.