ECE 5273
Test 2

Wednesday, April 8, 1998, 4:30 PM - 5:45 PM

Spring 1998 Name: SOLUTION

Dr. Havlicek Student Num:__________

Directions: There are **seven** problems on this test. You have 75 minutes to complete the test. All work must be your own. You may use two 8.5 × 11 inch two-sided note sheets.

All Students: Work problems 1 and 2.

Students enrolled for undergraduate credit: Work **three** problems out of problems 3 through 7. Each of these problems counts **20 pts**. Below, circle the numbers of the **three** problems you wish to have graded.

Students enrolled for graduate credit: Work **four** problems out of problems 3 through 7. Each of these problems counts **15 pts**. Below, circle the numbers of the **four** problems you wish to have graded.

SHOW ALL OF YOUR WORK for maximum partial credit! Good Luck!

Circle the numbers of the problems you wish to have graded:

1. **2.** **3.** **4.** **5.** **6.** **7.**

SCORE:

1.	(25)	______	4.	(20/15)	______	7.	(20/15)	______
2.	(15)	______	5.	(20/15)	______			
3.	(20/15)	______	6.	(20/15)	______			

TOTAL (100):

1. **25 pts.** True or False. Mark the correct answer. Mark *True* only if the statement is **always** true.

TRUE	FALSE	
X		(a) **2 pts.** If two images are discrete and periodic with the same period, then their wraparound convolution equals their linear convolution.
X		(b) **2 pts.** If $\mathbb{I}$ is a $N \times N$ image with DFT $\tilde{\mathbb{I}}$ and $\mathbb{J}$ is a $N \times N$ image with DFT $\tilde{\mathbb{J}}$, then the pointwise product $\tilde{\mathbb{I}} \otimes \tilde{\mathbb{J}}$ is $1/N$ times the DFT of the wraparound convolution of $\mathbb{I}$ and $\mathbb{J}$.
X		(c) **2 pts.** With appropriate zero padding, wraparound convolution can be used to implement linear convolution.
	X	(d) **2 pts.** When a linear filter is used to smooth white noise, it is usually a **high pass** filter.
	X	(e) **2 pts.** Streaking and blotching are artifacts that can occur when a median filter is used with a window (structuring element) $\mathbb{B}$ that is too **small**.
X		(f) **2 pts.** A **linear translation invariant** digital image processing system can be completely characterized by the system unit pulse response.
X		(g) **2 pts.** It is **always** possible to define a bandpass filter by taking the difference of two low-pass filters that are identical except for a scaling factor.
	X	(h) **2 pts.** **Nonlinear** filtering is based on frequency or spectrum shaping.
	X	(i) **2 pts.** The DFT is one of the most useful tools for analyzing the operation of nonlinear filters.
X		(j) **2 pts.** The usual convention for handling edge effects in nonlinear filtering is **replication**.
	X	(j) **2 pts.** The median filter removes positive impulses while preserving negative impulses.
	X	(l) **3 pts.** The instructor used to date Lena.

2. **15 pts**. Short answer.

(a) **3 pts**. A desirable property of the MED, OPEN-CLOSE, and CLOSE-OPEN filters is that they can retain important image structures. Name a disadvantage of these filters.

They can produce streaking and blotching effects which can appear as artifacts.

(b) **3 pts**. Suppose that $\mathbb{I}$ is an image. Write an equation that describes a morphological image processing operation to detect **peaks** in $\mathbb{I}$.

$\mathbb{J}_{\text{peak}} = \mathbb{I} - \mathbb{I} \circ \mathbb{B}$

(c) **3 pts**. Name two typical uses for for **low-pass** filters.

1. Smooth noise
2. Blur image details to emphasize gross features.

(d) **3 pts**. Give the frequency response for a **high-pass** digital filter that approximates the **continuous** Laplacian.

$$\tilde{H}(u,v) = A(u^2+v^2)/N^2 \ ; \quad 0 \leq |u|, |v| \leq \frac{N}{2} - 1$$

(e) **3 pts**. State one of the two constraints we used to regularize the ill-posed problem of restoring an image that has been blurred and corrupted by additive white noise.

The data constraint: minimize $\| \mathbb{G} * \hat{\mathbb{I}} - \mathbb{J} \|$.

The smoothness constraint: minimize $\| \nabla^2 \hat{\mathbb{I}} \|$.

3. **UG 20 pts. / G 15 pts.** Consider a linear digital image processing system with frequency response

$$\widetilde{\mathbb{H}} = \left[\widetilde{H}(u,v)\right].$$

Suppose that the system input is the $N \times N$ image $\mathbb{I}$ defined by

$$I(i,j) = \cos\left[\frac{2\pi}{N}(bi + cj)\right].$$

Find the system response $J(i,j) = H(i,j) * I(i,j)$.

$$J(i,j) = \left|\widetilde{H}(b,c)\right| \cos\left[\frac{2\pi}{N}(bi + cj) + \measuredangle \widetilde{H}(b,c)\right]$$

4. **UG 20 pts. / G 15 pts.** Pixels in the 4×4 image $\mathbb{I}$ shown below take gray levels in the range $\{0, 1, 2, \ldots, 99\}$. The image is sent through a communication channel where it is corrupted by noise. The received image $\mathbb{J}$ is shown below.

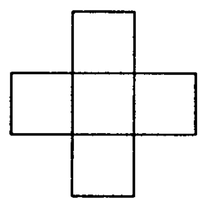
$\mathbb{I} =$

72	**72**	**73**	**74**
72	**99**	**72**	**74**
74	**75**	**71**	**70**
75	**71**	**69**	**69**

$\mathbb{J} =$

72	**72**	**73**	**74**
72	**99**	**72**	**74**
74	**75**	**0**	**70**
75	**71**	**69**	**69**

Choose an appropriate morphological operation to restore the received image by attenuating the noise. Use the window (structuring element) $\mathbb{B} = \text{CROSS}(5)$:



Handle edge effects by replication. Show the restored image $\hat{\mathbb{I}}$ below. You may use page 6 for work space.

The image contains a positive-going impulse.

The noise introduces a negative-going impulse.

$\Longrightarrow$ USE CLOSE.

Show the restored image here:

$\hat{\mathbb{I}} =$

72	72	74	74
72	99	74	74
75	75	71	70
75	71	70	70

work on page 6 $\longrightarrow$

More work space for problem 4...

$\mathbb{K} = \text{DILATE}[\mathbb{J}]$

72	99	74	74
99	99	99	74
75	99	75	74
75	75	71	70

$\hat{\mathbb{I}} = \text{ERODE}[\mathbb{K}] = \text{CLOSE}[\mathbb{J}]$

72	72	74	74
72	99	74	74
75	75	71	70
75	71	70	70

5. **UG 20 pts. / G 15 pts.** The $N \times N$ digital image $\mathbb{I}_1$ is defined by

$$I_1(i,j) = 5 + 3\delta(i,j)$$

and the $N \times N$ digital image $\mathbb{I}_2$ is defined by

$$I_2(i,j) = 1 + 6\cos\left[\frac{2\pi}{N}(2i + 7j)\right].$$

Let the $N \times N$ digital image $\mathbb{J}$ be the wraparound convolution of $\mathbb{I}_1$ and $\mathbb{I}_2$. Find $\tilde{\mathbb{J}}$, the DFT of $\mathbb{J}$.

$$\tilde{\mathbb{I}}_1 = 5N\delta(u,v) + \frac{3}{N}$$

$$\tilde{\mathbb{I}}_2 = N\delta(u,v) + \frac{6N}{2}\left[\delta(u-2,v-7) + \delta(u+2,v+7)\right]$$

$$\tilde{\mathbb{J}} = \mathrm{DFT}\left[\mathbb{I}_1 \circledast \mathbb{I}_2\right] = N\tilde{\mathbb{I}}_1\tilde{\mathbb{I}}_2$$

$$= N\Big\{5N^2\delta(u,v) + \cancelto{0}{5N\delta(u,v)\frac{6N}{2}\left[\delta(u-2,v-7)+\delta(u+2,v+7)\right]} + 3\delta(u,v)$$

$$+\frac{18N}{2N}\left[\delta(u-2,v-7)+\delta(u+2,v+7)\right]\Big\}$$

$$= 5N^3\delta(u,v) + 3N\delta(u,v) + 9N\left[\delta(u-2,v-7)+\delta(u+2,v+7)\right]$$

$$\boxed{\tilde{\mathbb{J}} = \left[5N^3 + 3N\right]\delta(u,v) + 9N\left[\delta(u-2,v-7)+\delta(u+2,v+7)\right]}$$

6. **UG 20 pts. / G 15 pts.** Consider the 2×2 images $\mathbb{I}_1$ and $\mathbb{I}_2$ shown below. Pixels in these images take gray levels in the range $\{0, 1, 2, \ldots, 15\}$. Pixel $I_1(i, j)$ is the pixel in ROW i and COLUMN j of image $\mathbb{I}_1$.

$$\mathbb{I}_1 = \begin{array}{|c|c|}\hline \mathbf{1} & \mathbf{2} \\ \hline \mathbf{0} & \mathbf{1} \\ \hline \end{array} \qquad \mathbb{I}_2 = \begin{array}{|c|c|}\hline \mathbf{3} & \mathbf{1} \\ \hline \mathbf{4} & \mathbf{1} \\ \hline \end{array}$$

Let the image $\mathbb{J} = \mathbb{I}_1 * \mathbb{I}_2$ be the **linear** convolution of $\mathbb{I}_1$ and $\mathbb{I}_2$. Compute the image $\mathbb{J}$ directly. Show your answer below.

$$J(i,j) = \sum_{m=0}^{1} \sum_{n=0}^{1} I_1(m,n)\, I_2(i-m, j-n)$$

$$J(0,0) = I_1(0,0)I_2(0,0) + I_1(0,1)I_2(0,-1) + I_1(1,0)I_2(-1,0) + I_1(1,1)I_2(-1,-1)$$
$$= 3 + 0 + 0 + 0 = 3$$

$$J(0,1) = I_1(0,0)I_2(0,1) + I_1(0,1)I_2(0,0) + I_1(1,0)I_2(-1,1) + I_1(1,1)I_2(-1,0)$$
$$= 1 + 6 + 0 + 0 = 7$$

$$J(1,0) = I_1(0,0)I_2(1,0) + I_1(0,1)I_2(1,-1) + I_1(1,0)I_2(0,0) + I_1(1,1)I_2(0,-1)$$
$$= 4 + 0 + 0 + 0 = 4$$

$$J(1,1) = I_1(0,0)I_2(1,1) + I_1(0,1)I_2(1,0) + I_1(1,0)I_2(0,1) + I_1(1,1)I_2(0,0)$$
$$= 1 + 8 + 0 + 3 = 12$$

Show the result image here:

$$\mathbb{J} = \begin{array}{|c|c|}\hline 3 & 7 \\ \hline 4 & 12 \\ \hline \end{array}$$

7. **UG 20 pts. / G 15 pts.** Pixels in the 4×4 image $\mathbb{I}$ shown below take gray levels in the range $\{0, 1, 2, \ldots, 99\}$. The image is sent through a communication channel where it is corrupted by noise. The received image $\mathbb{J}$ is shown below.

$\mathbb{I} =$

72	**72**	**10**	**10**
72	**72**	**10**	**10**
72	**72**	**10**	**10**
72	**72**	**10**	**10**

$\mathbb{J} =$

65	**72**	**10**	**10**
72	**73**	**10**	**10**
72	**72**	**22**	**10**
72	**75**	**10**	**10**

Use a median filter to restore the received image by attenuating the noise. For the window (structuring element), use $\mathbb{B} = \text{ROW}(3) = \{(0,-1),(0,0),(0,1)\}$. Handle edge effects by replication. Show the restored image $\mathbb{K}$ below. Also, find the *ISNR* (improvement in signal to noise ratio) for the restored image $\mathbb{K}$ relative to the received image $\mathbb{J}$. You may use page 10 for work space.

$\mathbb{B} =$

$$\boxed{\text{ISNR} = -0.7632 \text{ dB}}$$

see work on page 10.

$\Rightarrow$ In this case, MF makes it worse!!

Show the restored image here:

$\mathbb{K} =$

65	65	10	10
72	72	10	10
72	72	22	10
72	72	10	10

More work space for problem 7...

$$MSE(\mathbb{J}) = \frac{49+1+144+9}{16} = \frac{203}{16} = 12.6875$$

$$MSE(\mathbb{K}) = \frac{49+49+144}{16} = \frac{242}{16} = 15.125$$

$$ISNR = 10\log_{10}\frac{MSE(\mathbb{J})}{MSE(\mathbb{K})} = -0.7632\ dB$$

$|\mathbb{K}-\mathbb{I}| / (\mathbb{K}-\mathbb{I})^2$

7/49	7/49	0/0	0/0
0/0	0/0	0/0	0/0
0/0	0/0	12/144	0/0
0/0	0/0	0/0	0/0

$\mathbb{K} = MF[\mathbb{J}, \mathbb{B}]$

65	65	10	10
72	72	10	10
72	72	22	10
72	72	10	10

$|\mathbb{J}-\mathbb{I}| / (\mathbb{J}-\mathbb{I})^2$

7/49	0/0	0/0	0/0
0/0	1/1	0/0	0/0
0/0	0/0	12/144	0/0
0/0	3/9	0/0	0/0
