ECE 2713
Test 1

Thursday, March 26, 2026
12:00 PM - 1:15 PM

Spring 2026

Dr. Havlicek

Name: SOLUTION

Student Num: ____________

Directions: This test is closed book and closed notes. Calculators are allowed. You may also use the formula sheet provided with the test. All work must be your own. You have 75 minutes to complete the test.

SHOW ALL OF YOUR WORK for maximum partial credit!

GOOD LUCK!

SCORE:

1. (25) ________

2. (25) ________

3. (25) ________

4. (25) ________

____

TOTAL (100):

____

On my honor, I affirm that I have neither given nor received inappropriate aid in the completion of this test.

Name: ____________________ Date: ____________________

1. **25 pts**. Consider the discrete-time signal

$$\omega_0 = \frac{12\pi}{62}$$

$$x[n] = 5\cos\left(\frac{12\pi}{62}n\right).$$

(a) **13 pts**. Is $x[n]$ periodic? If you say *no*, then explain why not. If you say *yes*, then find the fundamental period.

$$\frac{\omega_0}{2\pi} = \frac{12\pi}{62}\cdot\frac{1}{2\pi} = \frac{6}{62} = \frac{3}{31} \in \mathbb{Q} \Rightarrow$$

$\boxed{x[n] \text{ is periodic}}$

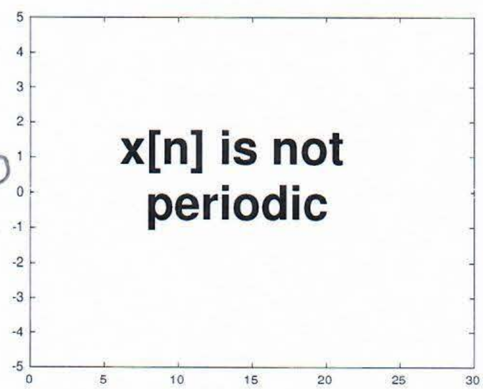
$$\frac{\omega_0}{2\pi} = \qquad = \frac{3}{31} = \frac{m}{N}$$

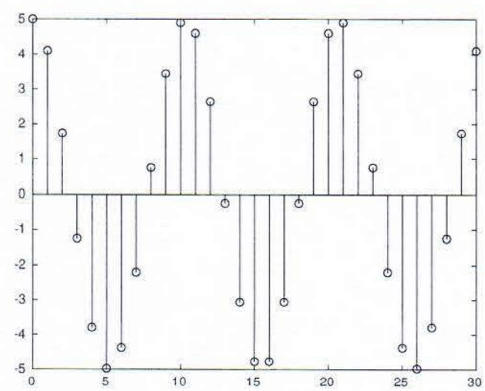
$\boxed{\text{Fundamental Period} = N = 31}$

$m = 3$: graph "goes around" 3 times to make one period.

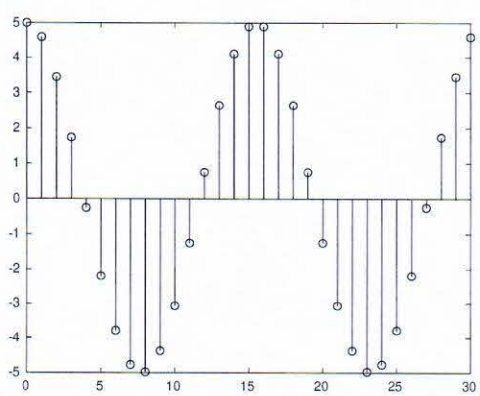
(b) **12 pts**. Circle the graph that shows one period of $x[n]$:

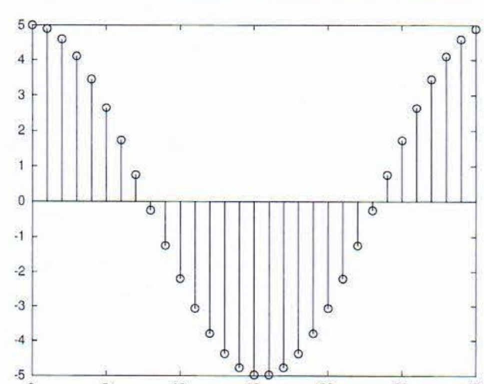
X because $x[n]$ is periodic





X because this graph only goes around 2 times





X because this graph only goes around one time

2. **25 pts**. A continuous-time signal $x(t)$ is given by

$$x(t) = 4\cos\left(100t + \frac{2\pi}{5}\right) + 12\cos\left(100t + \frac{5\pi}{9}\right).$$

Use phasor addition to express $x(t)$ in the form

$$x(t) = A\cos(100t + \phi).$$

Phasor for $4\cos\left[100t + \frac{2\pi}{5}\right]$: $X_1 = 4e^{j2\pi/5}$

Phasor for $12\cos\left[100t + 5\pi/9\right]$: $X_2 = 12e^{j5\pi/9}$

Phasor for $x(t)$:

$$X = X_1 + X_2 = 4e^{j2\pi/5} + 12e^{j5\pi/9}$$

$$= 4\cos\frac{2\pi}{5} + j4\sin\frac{2\pi}{5} + 12\cos\frac{5\pi}{9} + j12\sin 5\pi/9$$

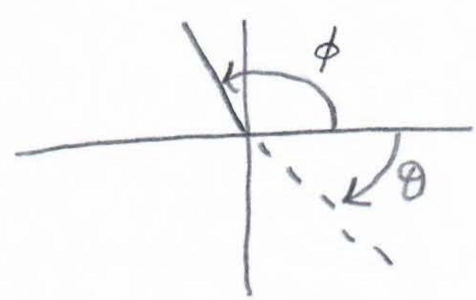
$$= 1.23607 + j3.80423 + (-2.08378) + j11.8177$$

$$= -0.847710 + j15.6219$$

$$|X| = \sqrt{(-0.847710)^2 + (15.6219)^2}$$

$$= \sqrt{244.763} = 15.645$$

$$\phi = \arg X = \arctan\left[\frac{15.6219}{-0.847710}\right]$$



Since ϕ is second quadrant, atan will give θ instead. Need to add or subtract π to get ϕ.

$$\theta = \text{atan}\left[\frac{15.6219}{-0.84771}\right] = -1.51659$$

$$\phi = \theta + \pi = 1.62501$$

$$X = |X|e^{j\phi} = 15.645e^{j1.625}$$

$$\boxed{x(t) = 15.645\cos(100t + 1.625)}$$

More Workspace for Problem 2...

3. **25 pts**. A discrete-time LTI system H has impulse response $h[n]$ given by

$$h[n] = \frac{1}{3}\delta[n] - \frac{2}{3}\delta[n-1] + \frac{1}{3}\delta[n-2].$$

The system input is given by

$$x[n] = 3\delta[n+1] + 6\delta[n-1].$$

Find the system output $y[n]$.

$$y[n] = x[n] * h[n] = 3\delta[n+1] * h[n] + 6\delta[n-1] * h[n]$$

$$= 3h[n+1] + 6h[n-1]$$

$$= 3\left\{\tfrac{1}{3}\delta[n+1] - \tfrac{2}{3}\delta[n] + \tfrac{1}{3}\delta[n-1]\right\}$$

$$+6\left\{\tfrac{1}{3}\delta[n-1] - \tfrac{2}{3}\delta[n-2] + \tfrac{1}{3}\delta[n-3]\right\}$$

$$= \delta[n+1] - 2\delta[n] + \delta[n-1]$$

$$+ 2\delta[n-1] - 4\delta[n-2] + 2\delta[n-3]$$

$$\boxed{y[n] = \delta[n+1] - 2\delta[n] + 3\delta[n-1] - 4\delta[n-2] + 2\delta[n-3]}$$

OTHER WAY: $y[n] = h[n] * x[n] = \left\{\tfrac{1}{3}\delta[n] - \tfrac{2}{3}\delta[n-1] + \tfrac{1}{3}\delta[n-2]\right\} * x[n]$

$$= \tfrac{1}{3}\delta[n] * x[n] - \tfrac{2}{3}\delta[n-1] * x[n] + \tfrac{1}{3}\delta[n-2] * x[n]$$

$$= \tfrac{1}{3}x[n] - \tfrac{2}{3}x[n-1] + \tfrac{1}{3}x[n-2]$$

$$= \delta[n+1] + 2\delta[n-1]$$

$$- 2\delta[n] - 4\delta[n-2]$$

$$+ \delta[n-1] + 2\delta[n-3]$$

$$\boxed{y[n] = \delta[n+1] - 2\delta[n] + 3\delta[n-1] - 4\delta[n-2] + 2\delta[n-3]}$$

4. **25 pts**. A discrete-time LTI system H has impulse response $h[n]$ given by

$$h[n] = \left(\frac{1}{4}\right)^n (u[n] - u[n-4]) = \begin{cases} \left(\frac{1}{4}\right)^n, & 0 \le n \le 3, \\ 0, & \text{otherwise.} \end{cases}$$

The system input is given by

$$x[n] = \left(\frac{1}{2}\right)^n u[n+1].$$

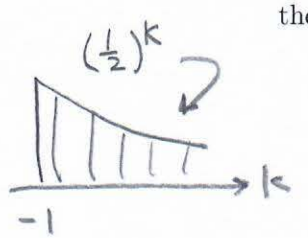
Find the system output $y[n]$.

Hint: here are the steps for performing convolution:

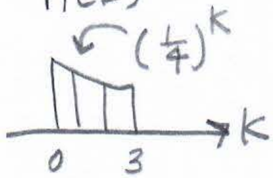
1. write $y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$.
2. Use the definition of $x[n]$ given above to draw the graph of $x[k]$.
3. Use the definition of $h[n]$ given above to draw the graph of $h[k]$.
4. Slide the graph of $h[k]$ to the right by $-n$ to get the graph of $h[k-(-n)] = h[n+k]$.
5. Flip the graph of $h[n+k]$ with respect to k to get the graph of $h[-k-(-n)] = h[n-k]$.
6. For the n's in each region, multiply the graph of $x[k]$ with the graph of $h[n-k]$ and add up the product graph to get $y[n]$.

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

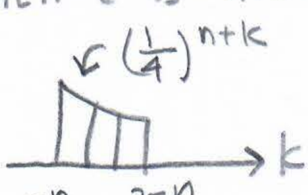
$x[k]$



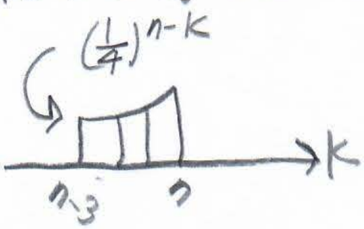
$h[k]$



$h[k-(-n)] = h[n+k]$

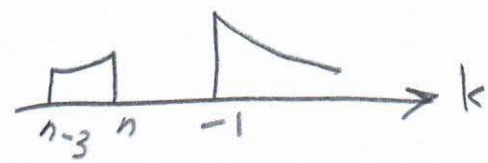


$h[-k-(-n)] = h[n-k]$

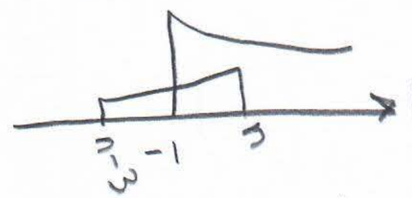


case I) $n < -1$

$$y[n] = \sum_{k=-\infty}^{\infty} 0 = 0$$



case II) $n \geq -1$ and $n-3 < -1$: $-1 \le n < 2$



$$y[n] = \sum_{k=-1}^{n} \left(\tfrac{1}{2}\right)^k \left(\tfrac{1}{4}\right)^{n-k}$$

$$= \sum_{k=-1}^{n} \left(\tfrac{1}{2}\right)^k \left(\tfrac{1}{4}\right)^n \left(\tfrac{1}{4}\right)^{-k}$$

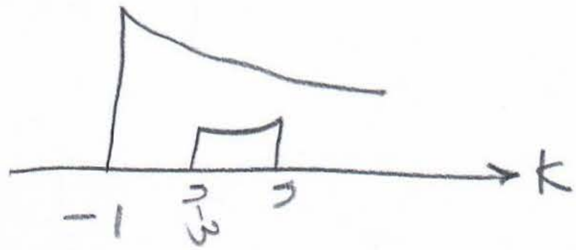
$$= \left(\tfrac{1}{4}\right)^n \sum_{k=-1}^{n} \left(\tfrac{1}{2}\right)^k 4^k = \left(\tfrac{1}{4}\right)^n \sum_{k=-1}^{n} \left(\tfrac{4}{2}\right)^k = \left(\tfrac{1}{4}\right)^n \sum_{k=-1}^{n} 2^k$$

$$= \left(\tfrac{1}{4}\right)^n \frac{2^{-1} - 2^{n+1}}{1-2} = \left(\tfrac{1}{4}\right)^n \frac{\frac{1}{2} - 2 \cdot 2^n}{-1}$$

$$= \left(\tfrac{1}{4}\right)^n \left[2 \cdot 2^n - \tfrac{1}{2}\right] = 2\left(\tfrac{1}{4}\right)^n 2^n - \tfrac{1}{2}\left(\tfrac{1}{4}\right)^n$$

$$= 2\left(\tfrac{2}{4}\right)^n - \tfrac{1}{2}\left(\tfrac{1}{4}\right)^n$$

$$= 2\left(\tfrac{1}{2}\right)^n - \tfrac{1}{2}\left(\tfrac{1}{4}\right)^n \quad \mapsto$$



case III) $n \geq 2$

$$y[n] = \sum_{k=n-3}^{n} \left(\tfrac{1}{2}\right)^k \left(\tfrac{1}{4}\right)^{n-k}$$

$$= \left(\tfrac{1}{4}\right)^n \sum_{k=n-3}^{n} \left(\tfrac{1}{2}\right)^k \left(\tfrac{1}{4}\right)^{-k} = \left(\tfrac{1}{4}\right)^n \sum_{k=n-3}^{n} \left(\tfrac{1}{2}\right)^k 4^k = \left(\tfrac{1}{4}\right)^n \sum_{k=n-3}^{n} \left(\tfrac{4}{2}\right)^k$$

$$= \left(\tfrac{1}{4}\right)^n \sum_{k=n-3}^{n} 2^k = \left(\tfrac{1}{4}\right)^n \frac{2^{n-3} - 2^{n+1}}{1-2} = \left(\tfrac{1}{4}\right)^n \frac{2^{-3}\cdot 2^n - 2\cdot 2^n}{-1}$$

$$= \left(\tfrac{1}{4}\right)^n \left[2\cdot 2^n - 2^{-3}\cdot 2^n\right] = \left(\tfrac{1}{4}\right)^n 2^n \left[2 - \tfrac{1}{8}\right]$$

$$= \left[\tfrac{16}{8} - \tfrac{1}{8}\right] \left(\tfrac{2}{4}\right)^n = \tfrac{15}{8} \left(\tfrac{1}{2}\right)^n$$

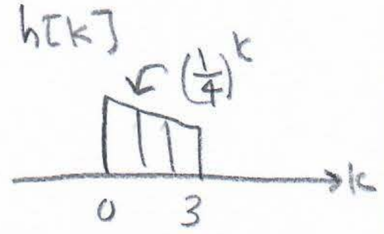
All Together:

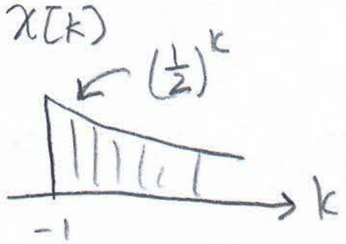
$$\boxed{y[n] = \begin{cases} 0, & n < -1 \\ 2\left(\tfrac{1}{2}\right)^n - \tfrac{1}{2}\left(\tfrac{1}{4}\right)^n, & -1 \leq n < 2 \\ \tfrac{15}{8}\left(\tfrac{1}{2}\right)^n, & n \geq 2 \end{cases}}$$

OTHER WAY...

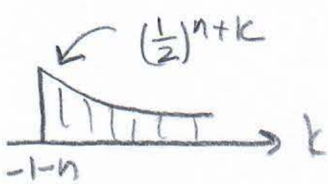
More Workspace for Problem 4...

$$y[n] = h[n] * x[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

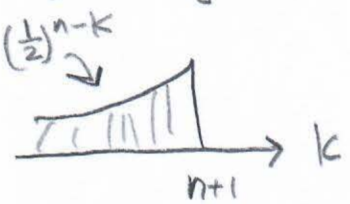




$x[k-(-n)] = x[n+k]$

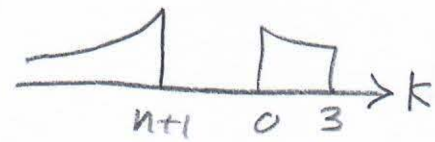


$x[-k-(-n)] = x[n-k]$



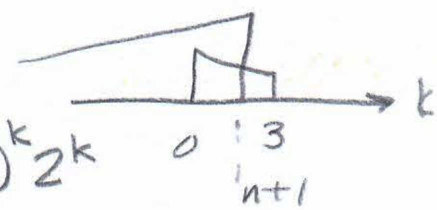
case I) $n+1 < 0$: $n < -1$

$$y[n] = \sum_{k=-\infty}^{\infty} 0 = 0$$



case II) $n+1 \geq 0$ and $n+1 < 3$: $-1 \leq n < 2$

$$y[n] = \sum_{k=0}^{n+1} \left(\tfrac{1}{4}\right)^k \left(\tfrac{1}{2}\right)^{n-k}$$



$$= \left(\tfrac{1}{2}\right)^n \sum_{k=0}^{n+1} \left(\tfrac{1}{4}\right)^k \left(\tfrac{1}{2}\right)^{-k} = \left(\tfrac{1}{2}\right)^n \sum_{k=0}^{n+1} \left(\tfrac{1}{4}\right)^k 2^k$$

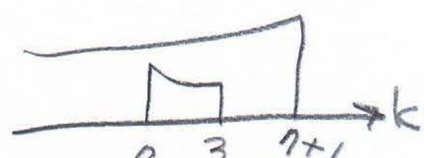
$$= \left(\tfrac{1}{2}\right)^n \sum_{k=0}^{n+1} \left(\tfrac{2}{4}\right)^k = \left(\tfrac{1}{2}\right)^n \sum_{k=0}^{n+1} \left(\tfrac{1}{2}\right)^k = \left(\tfrac{1}{2}\right)^n \frac{\left(\frac{1}{2}\right)^0 - \left(\frac{1}{2}\right)^{n+2}}{1-1/2}$$

$$= \left(\tfrac{1}{2}\right)^n \frac{1 - \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^n}{1/2} = 2\left(\tfrac{1}{2}\right)^n \left[1 - \tfrac{1}{4}\left(\tfrac{1}{2}\right)^n\right]$$

$$= 2\left(\tfrac{1}{2}\right)^n - \tfrac{2}{4}\left(\tfrac{1}{2}\right)^n \left(\tfrac{1}{2}\right)^n = 2\left(\tfrac{1}{2}\right)^n - \tfrac{1}{2}\left(\tfrac{1}{4}\right)^n$$

case III) $n+1 \geq 3$: $n \geq 2$

$$y[n] = \sum_{k=0}^{3} \left(\tfrac{1}{4}\right)^k \left(\tfrac{1}{2}\right)^{n-k}$$



$$= \left(\tfrac{1}{2}\right)^n \sum_{k=0}^{3} \left(\tfrac{1}{4}\right)^k \left(\tfrac{1}{2}\right)^{-k} = \left(\tfrac{1}{2}\right)^n \sum_{k=0}^{3} \left(\tfrac{1}{4}\right)^k 2^k = \left(\tfrac{1}{2}\right)^n \sum_{k=0}^{3} \left(\tfrac{2}{4}\right)^k$$

$$= \left(\tfrac{1}{2}\right)^n \sum_{k=0}^{3} \left(\tfrac{1}{2}\right)^k = \left(\tfrac{1}{2}\right)^n \frac{\left(\frac{1}{2}\right)^0 - \left(\frac{1}{2}\right)^4}{1-1/2} = \left(\tfrac{1}{2}\right)^n \frac{1 - 1/16}{1/2} = \left(\tfrac{1}{2}\right)^n \cdot 2 \cdot \tfrac{15}{16} = \tfrac{15}{8}\left(\tfrac{1}{2}\right)^n$$

All Together:

$$y[n] = \begin{cases} 0, & n < -1 \\ 2\left(\frac{1}{2}\right)^n - \frac{1}{2}\left(\frac{1}{4}\right)^n, & -1 \leq n < 2 \\ \frac{15}{8}\left(\frac{1}{2}\right)^n, & n \geq 2 \end{cases}$$