ECE 3793
Homework 4 Solution

Spring 2017

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ECE 3793 HW 4 Solution, Spring 2017, Havlicek

Problem 1)

2.14b)

$h_2(t) = e^{-t}\cos(2t)\,u(t).$

$$\int_{-\infty}^{\infty} |h_2(t)|\,dt = \int_{-\infty}^{\infty} \left| e^{-t}\cos(2t)\,u(t) \right| dt$$

$$= \int_0^{\infty} |e^{-t}\cos(2t)|\,dt = \int_0^{\infty} |e^{-t}|\,|\cos 2t|\,dt$$

$$\leq \int_0^{\infty} |e^{-t}| \cdot 1\,dt = \int_0^{\infty} e^{-t}\,dt = -\left[e^{-t}\right]_0^{\infty}$$

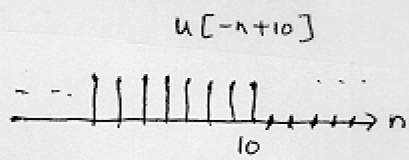
$$= -[0-1] = 1 < \infty.$$

Since $\int_{-\infty}^{\infty} |h_2(t)|\,dt < \infty$, the system is stable.

Problem 2)

2.15b)

$h_2[n] = 3^n u[-n+10]$



$$\sum_{n=-\infty}^{\infty} |h_2[n]| = \sum_{n=-\infty}^{\infty} \left| 3^n u[-n+10] \right|$$

$$= \sum_{n=-\infty}^{10} 3^n \qquad \begin{array}{l} m = -n+10 \\ n = -m+10 \end{array}$$

$$= \sum_{m=\infty}^{0} 3^{-m+10} = 3^{10}\sum_{m=0}^{\infty} 3^{-m} = 3^{10}\left[\frac{1}{1-\frac{1}{3}}\right]$$

$$= 3^{10}\left(\frac{3}{2}\right) = \frac{3^{11}}{2} < \infty$$

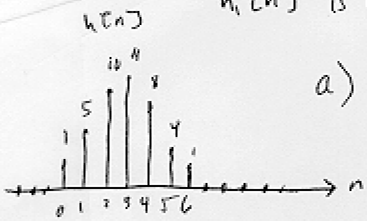
Therefore, the system is stable.

Problem 3)

2.24a)

$h_2[n] = u[n] - u[n-2] = \delta[n] + \delta[n-1]$.

$h_1[n]$ is unknown. But, H_1 is causal, so $h_1[n] = 0,\ n<0$.



a) Let $h_3[n] = h_2[n] * h_2[n]$.

Then $h_3[n] = \left(\delta[n] + \delta[n-1]\right) * \left(\delta[n] + \delta[n-1]\right)$

$= \delta[n] * \left(\delta[n] + \delta[n-1]\right)$
$+ \delta[n-1] * \left(\delta[n] + \delta[n-1]\right)$

$= \delta[n] + \delta[n-1] + \delta[n-1] + \delta[n-2]$

$= \delta[n] + 2\delta[n-1] + \delta[n-2]$.

Now, $h[n] = h_1[n] * \left[h_2[n] * h_2[n]\right]$

$= h_1[n] * h_3[n]$

Problem 3)…

$$\left(h[n]\right) = h_1[n] * \left(\delta[n] + 2\delta[n-1] + \delta[n-2]\right)$$

$$\underline{h[n] = h_1[n] + 2h_1[n-1] + h_1[n-2]}$$

$\underline{n=0}$ $$h[0] = 1 = h_1[0] + \cancelto{0}{2h_1[-1]} + \cancelto{0}{h_1[-2]} = h_1[0]$$

$$\Rightarrow h_1[0] = 1$$

$\underline{n=1}$ $$h[1] = 5 = h_1[1] + 2h_1[0] + \cancelto{0}{h_1[-1]}$$

$$= h_1[1] + 2\cdot 1 = h_1[1] + 2$$

$$\Rightarrow h_1[1] = 3$$

$\underline{n=2}$ $$h[2] = 10 = h_1[2] + 2h_1[1] + h_1[0]$$

$$= h_1[2] + 2\cdot 3 + 1 = h_1[2] + 7$$

$$\Rightarrow h_1[2] = 3$$

$\underline{n=3}$ $$h[3] = 11 = h_1[3] + 2h_1[2] + h_1[1]$$

$$= h_1[3] + 2\cdot 3 + 3 = h_1[3] + 9$$

$$\Rightarrow h_1[3] = 2$$

$\underline{n=4}$ $$h[4] = 8 = h_1[4] + 2h_1[3] + h_1[2]$$

$$= h_1[4] + 2\cdot 2 + 3 = h_1[4] + 7$$

$$\Rightarrow h_1[4] = 1$$

$$\longrightarrow$$

Problem 3)…

$\underline{n=5}$ $h[5] = 4 = h_1[5] + 2h_1[4] + h_1[3]$

$= h_1[5] + 2\cdot 1 + 2 = h_1[5] + 4$

$\Rightarrow h_1[5] = 0.$

$\underline{n=6}$ $h[6] = 1 = h_1[6] + 2h_1[5] + h_1[4]$

$= h_1[6] + 2\cdot 0 + 1 = h_1[6] + 1$

$\Rightarrow h_1[6] = 0$

$\underline{n>6}$ $h[n] = 0 = h_1[n] + 2h_1[n-1] + h_1[n-2]$

$= h_1[n] + 2\cdot 0 + 0 = h_1[n] + 0$

$\Rightarrow h_1[n] = 0.$

So $h_1[n] = \delta[n] + 3\delta[n-1] + 3\delta[n-2] + 2\delta[n-3] + \delta[n-4]$

2.24b) $h_1[n] = \delta[n] + 3\delta[n-1] + 3\delta[n-2] + 2\delta[n-3] + \delta[n-4]$

when $x[n] = \delta[n] - \delta[n-1]$,

$y[n] = x[n] * h[n] = (\delta[n] - \delta[n-1]) * h[n]$

$= \delta[n] * h[n] - \delta[n-1] * h[n]$

$= h[n] - h[n-1]$

$= \delta[n] + 4\delta[n-1] + 5\delta[n-2] + \delta[n-3] - 3\delta[n-4]$

$- 4\delta[n-5] - 3\delta[n-6]$

$- \delta[n-7]$

Problem 4)

2.28a) $h[n] = \left(\frac{1}{5}\right)^n u[n]$

$\Rightarrow h[n] = 0 \;\; \forall n < 0 \longrightarrow$ system <u>is</u> causal.

$$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=0}^{\infty} \left(\frac{1}{5}\right)^n = \frac{1}{1-\frac{1}{5}} = \frac{5}{4} < \infty.$$

$\longrightarrow$ System <u>is</u> stable.

2.28e) $h[n] = \left(-\frac{1}{2}\right)^n u[n] + (1.01)^n u[n-1]$

$h[n] = 0 \;\; \forall n < 0 \longrightarrow$ system <u>is</u> causal

$$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=0}^{\infty} \left| \left(-\frac{1}{2}\right)^n u[n] + (1.01)^n u[n-1] \right|$$

$$\geq \sum_{n=1}^{\infty} \left| \left(-\frac{1}{2}\right)^n + (1.01)^n \right|$$

$\geq$ Sum of terms from $n = 2$ to ∞ where n is even

$$= \sum_{n=2}^{\infty} \left| \left(-\frac{1}{2}\right)^{2n} + (1.01)^{2n} \right|$$

$$= \sum_{n=2}^{\infty} \left| \left(\frac{1}{2}\right)^{2n} + (1.01)^{2n} \right|$$

$$\geq \sum_{n=2}^{\infty} (1.01)^{2n} \longrightarrow \infty.$$

Since $\sum_{n=-\infty}^{\infty} |h[n]| \to \infty$, the system is <u>not</u> stable.

Problem 4)…

2.28g)

$h[n] = n\left(\frac{1}{3}\right)^n u[n-1]$

Causal, because $h[n] = 0 \quad \forall\ n < 0$.

$$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=-\infty}^{\infty} \left| n\left(\tfrac{1}{3}\right)^n u[n-1] \right| = \sum_{n=1}^{\infty} n\left(\tfrac{1}{3}\right)^n \qquad \begin{aligned} m &= n-1 \\ n &= m+1 \end{aligned}$$

$$= \sum_{m=0}^{\infty} (m+1)\left(\tfrac{1}{3}\right)^{m+1} = \frac{1}{3}\left[\sum_{m=0}^{\infty} m\left(\tfrac{1}{3}\right)^m + \sum_{m=0}^{\infty}\left(\tfrac{1}{3}\right)^m\right]$$

$$= \frac{1}{3}\left[\frac{1}{(1-\frac{1}{3})^2}\,\frac{1}{3} + \frac{1}{1-\frac{1}{3}}\right] = \frac{1}{3}\left[\frac{\frac{1}{3}}{4/9} + \frac{1}{2/3}\right]$$

$$= \frac{1}{3}\left[\frac{9}{12} + \frac{3}{2}\right] = \frac{1}{3}\left[\frac{9}{12} + \frac{18}{12}\right] = \frac{1}{3}\left[\frac{27}{12}\right]$$

$$= \frac{9}{12} = \frac{3}{4} < \infty$$

STABLE.

Problem 5)

2.29b)

$$h(t) = e^{-6t}\, u(3-t)$$

$\underline{\text{Not}}$ causal because $h(-1) = e^{6} \neq 0$.

$$\int_{-\infty}^{\infty} |h(t)|\,dt = \int_{-\infty}^{\infty} \left| e^{-6t} u(3-t) \right| dt = \int_{-\infty}^{3} e^{-6t}\, dt$$

$$= -\tfrac{1}{6}\left[e^{-6t}\right]_{-\infty}^{3} = \lim_{A\to\infty} -\tfrac{1}{6}\left[e^{-6t}\right]_{-A}^{3}$$

$$= \lim_{A\to\infty} -\tfrac{1}{6}\left[e^{-3t} - e^{At}\right] = \lim_{A\to\infty} \tfrac{1}{6}\left[e^{At} - e^{-3t}\right] = \infty$$

$\underline{\text{Unstable}}$.

2.29e)

$$h(t) = e^{-6|t|}$$

$\underline{\text{Not causal}}$ because $h(-1) = e^{-6} \neq 0$.

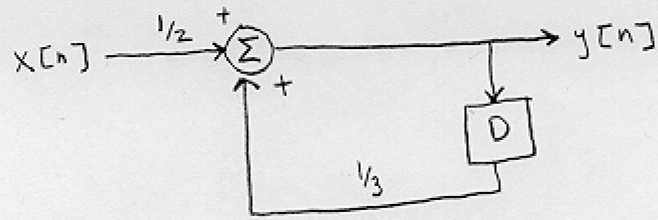
$$\int_{-\infty}^{\infty} |h(t)|\,dt = \int_{-\infty}^{\infty} |e^{-6|t|}|\,dt = \int_{-\infty}^{0} e^{6t}\,dt + \int_{0}^{\infty} e^{-6t}\,dt$$

$$= \tfrac{1}{6}e^{6t}\Big|_{-\infty}^{0} - \tfrac{1}{6}e^{-6t}\Big|_{0}^{\infty} = \tfrac{1}{6} - 0 - 0 + \tfrac{1}{6} = \tfrac{1}{3} < \infty.$$

The system $\underline{\text{is}}$ stable because $\int_{-\infty}^{\infty} |h(t)|\,dt < \infty$.

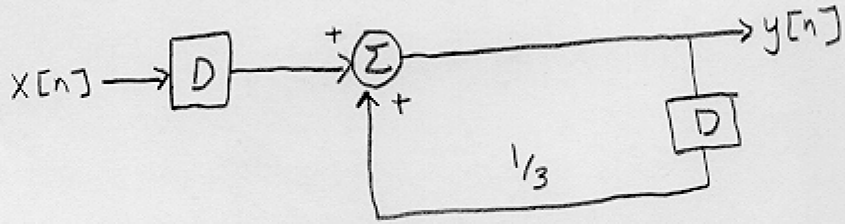
Problem 6)

2.38a) $y[n] = \frac{1}{3} y[n-1] + \frac{1}{2} x[n]$



2.38b)

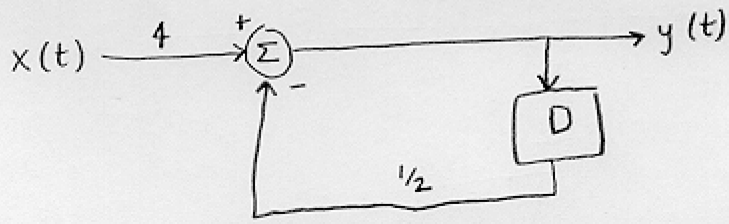
$y[n] = \frac{1}{3} y[n-1] + x[n-1]$



Problem 7)

2.39a)

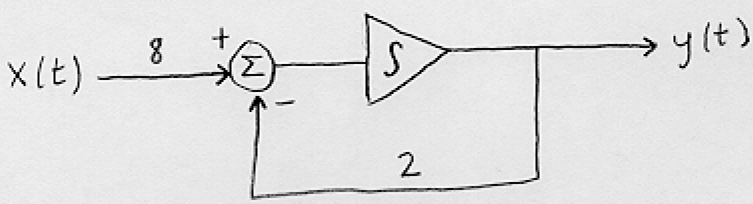
$$y(t) = -\tfrac{1}{2} y'(t) + 4x(t)$$



Alternatively, solving for $y'(t)$, we have

$$y'(t) = 8x(t) - 2y(t),$$

or $$y(t) = \int_{-\infty}^{t} 8x(\tau) - 2y(\tau)\, d\tau$$



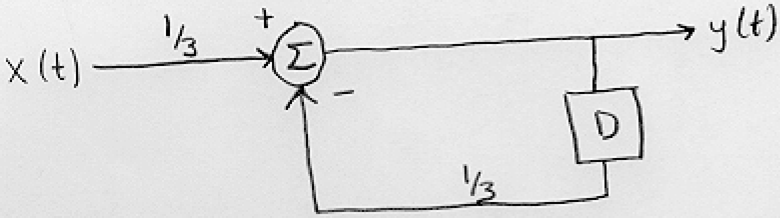
Problem 7)…

2.39b)

$$y'(t) + 3y(t) = x(t)$$

Solving for $y(t)$, we get

$$y(t) = \frac{1}{3}x(t) - \frac{1}{3}y'(t)$$

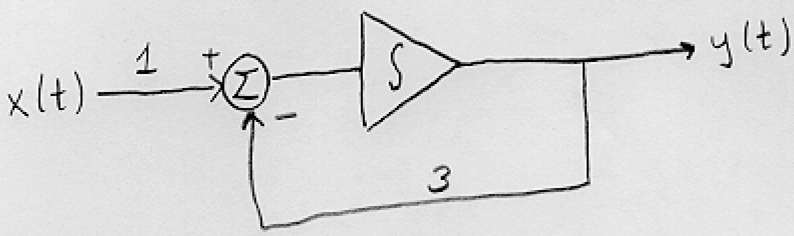


Alternatively, solving for $y'(t)$ gives us

$$y'(t) = x(t) - 3y(t),$$

or

$$y(t) = \int_{-\infty}^{t} x(\tau) - 3y(\tau)\, d\tau$$



Problem 8)

2.40a) $y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau-2)\, d\tau$

$h(t)$ is the output when $\delta(t)$ is the input:

$$h(t) = \int_{-\infty}^{t} e^{-(t-\tau)} \delta(\tau-2)\, d\tau$$

$$= \begin{cases} 0, & t < 2 \\ e^{-(t-2)}, & t \geq 2 \end{cases}$$

$$= e^{-(t-2)} u(t-2)$$