ECE 4213/5213
Test 1

Tuesday, November 10, 2026
4:30 PM - 5:45 PM

Fall 2026 Name: SOLUTION

Dr. Havlicek Student Num: _____

Directions: This test is open book and open notes. You may also use your calculator and the course formula sheet. Other materials are not allowed. You have 75 minutes to complete the test. All work must be your own.

Students enrolled for undergraduate credit: work any four problems. Each problem counts 25 points. Below, circle the numbers of the four problems you wish to have graded.

Students enrolled for graduate credit: work all five problems. Each problem counts 20 points.

SHOW ALL OF YOUR WORK for maximum partial credit!

GOOD LUCK!

SCORE:

1. (25/20) _____
2. (25/20) _____
3. (25/20) _____
4. (25/20) _____
5. (25/20) _____

TOTAL (100):

On my honor, I affirm that I have neither given nor received inappropriate aid in the completion of this test.

Signed: _____ Date: _____

1. **25/20 pts.** A causal, stable discrete-time LTI system H has impulse response $h[n]$ given by

$$h[n] = \left(\frac{1}{2}\right)^n u[n].$$

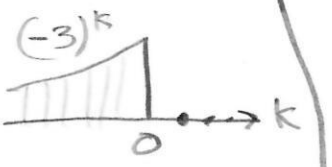
$y[n] = x[n] * h[n]$
$= \sum_{k \in \mathbb{Z}} x[k]h[n-k]$

The system input is given by

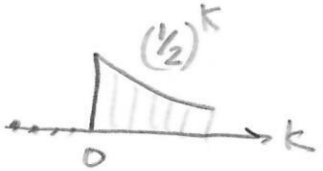
$$x[n] = \left(-\frac{1}{3}\right)^{-n} u[-n] \quad = \quad \begin{cases} (-3)^n, & n \leq 0, \\ 0, & \text{otherwise.} \end{cases}$$

(a) **13/10 pts.** Use time domain convolution to find the system output $y[n]$.

$x[k]$



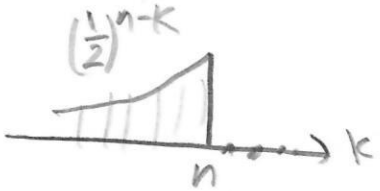
$h[k]$



$h[k--n] = h[n+k]$

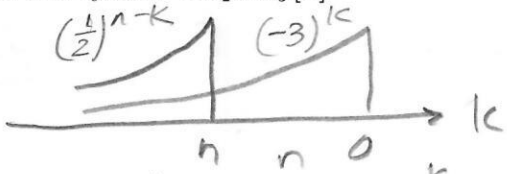


$h[-k--n] = h[n-k]$



Case I) $n < 0$

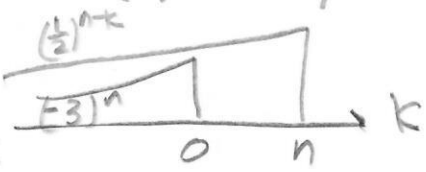
$$y[n] = \sum_{k=-\infty}^{n} \left(\tfrac{1}{2}\right)^{n-k}(-3)^k$$



$$= \left(\tfrac{1}{2}\right)^n \lim_{A \to -\infty} \sum_{k=A}^{n} 2^k(-3)^k = \left(\tfrac{1}{2}\right)^n \lim_{A \to -\infty} \sum_{k=A}^{n} (-6)^k$$

$$= \left(\tfrac{1}{2}\right)^n \lim_{A \to -\infty} \frac{-6^A - (-6)^{n+1}}{1+6} = \left(\tfrac{1}{2}\right)^n \frac{6(-6)^n}{7} = \frac{6}{7}(-3)^n$$

Case II) $n \geq 0$



$$y[n] = \sum_{k=-\infty}^{0} \left(\tfrac{1}{2}\right)^{n-k}(-3)^k$$

$$= \left(\tfrac{1}{2}\right)^n \lim_{A \to -\infty} \sum_{k=A}^{0} (-6)^k = \left(\tfrac{1}{2}\right)^n \lim_{A \to -\infty} \frac{(-6)^A - (-6)^1}{1+6}$$

$$= \left(\tfrac{1}{2}\right)^n \frac{6}{7} = \frac{6}{7}\left(\tfrac{1}{2}\right)^n$$

All Together:

$$y[n] = \begin{cases} \frac{6}{7}(-3)^n, & n < 0 \\ \frac{6}{7}\left(\frac{1}{2}\right)^n, & n \geq 0 \end{cases}$$

OTHER WAY

1. **25/20 pts.** A causal, stable discrete-time LTI system H has impulse response $h[n]$ given by

$$h[n] = \left(\frac{1}{2}\right)^n u[n].$$

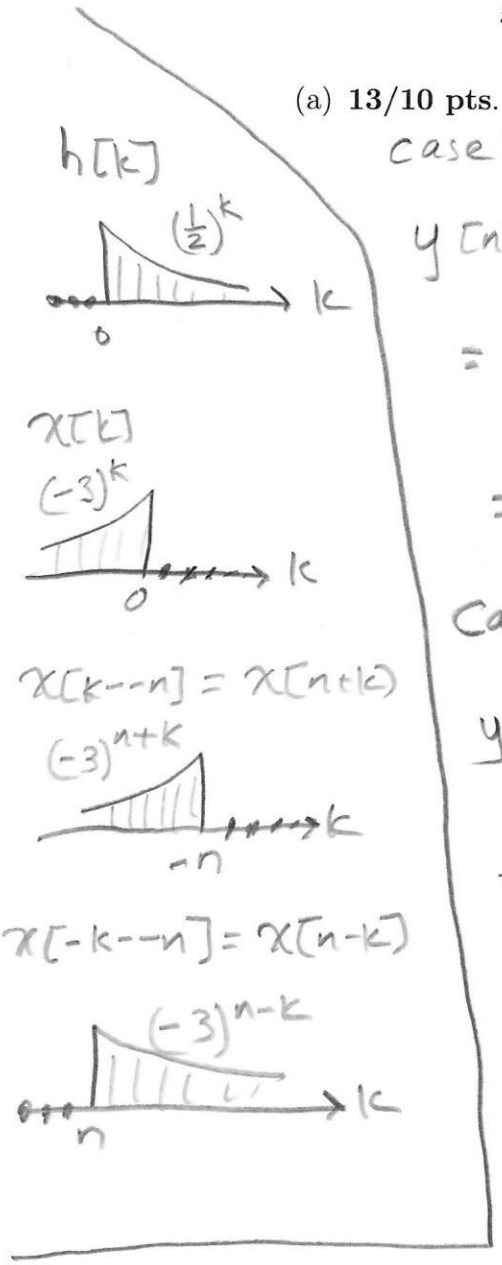
$y[n] = x[n] * h[n]$

$= \sum_{k \in \mathbb{Z}} h[k]\, x[n-k]$

The system input is given by

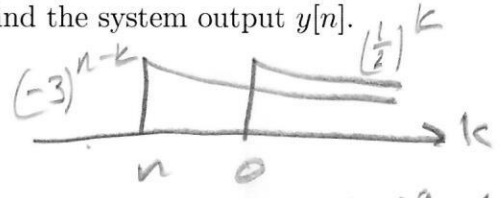
$$x[n] = \left(-\frac{1}{3}\right)^{-n} u[-n] \quad = \quad \begin{cases} (-3)^n, & n \le 0, \\ 0, & \text{otherwise.} \end{cases}$$

(a) **13/10 pts.** Use time domain convolution to find the system output $y[n]$.



case I) $n < 0$

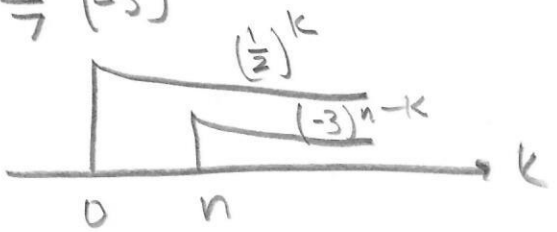
$$y[n] = \sum_{k=0}^{\infty} (-3)^{n-k} \left(\frac{1}{2}\right)^k$$



$$= (-3)^n \lim_{A \to \infty} \sum_{k=0}^{A} \left(-\frac{1}{6}\right)^k = (-3)^n \lim_{A \to \infty} \frac{\left(-\frac{1}{6}\right)^0 - \left(-\frac{1}{6}\right)^{A+1}}{1 + 1/6}$$

$$= (-3)^n \frac{1 - 0}{7/6} = \frac{6}{7}(-3)^n$$

Case II) $n \ge 0$



$$y[n] = \sum_{k=n}^{\infty} (-3)^{n-k} \left(\frac{1}{2}\right)^k$$

$$= (-3)^n \lim_{A \to \infty} \sum_{k=n}^{A} \left(-\frac{1}{6}\right)^k = (-3)^n \lim_{A \to \infty} \frac{\left(-\frac{1}{6}\right)^n - \left(-\frac{1}{6}\right)^{A+1}}{1 + \frac{1}{6}}$$

$$= (-3)^n \frac{(-1/6)^n}{7/6} = \frac{6}{7}\left(\frac{1}{2}\right)^n$$

All Together:

$$y[n] = \begin{cases} \frac{6}{7}(-3)^n, & n < 0 \\ \frac{6}{7}\left(\frac{1}{2}\right)^n, & n \ge 0 \end{cases}$$

Problem 1, cont...

(b) **12/10 pts.** Use the z-transform to find the system output $y[n]$.

Table: $H(z) = \dfrac{1}{1-\frac{1}{2}z^{-1}}$, $|z| > \frac{1}{2}$

Table: $(-\frac{1}{3})^n u[n] \overset{Z}{\longleftrightarrow} \dfrac{1}{1+\frac{1}{3}z^{-1}}$, $|z| > \frac{1}{3}$

Time Reversal:

$$(-\tfrac{1}{3})^{-n} u[-n] \overset{Z}{\longleftrightarrow} \frac{1}{1+\frac{1}{3}z} \cdot \frac{z^{-1}}{z^{-1}} = \frac{z^{-1}}{\frac{1}{3}+z^{-1}} \cdot \frac{3}{3}$$

$$= \frac{3z^{-1}}{1+3z^{-1}}, \quad |z| < 3$$

$$Y(z) = X(z)H(z) = \frac{3z^{-1}}{(1-\frac{1}{2}z^{-1})(1+3z^{-1})}, \quad \tfrac{1}{2} < |z| < 3$$

$$= \frac{A}{1-\frac{1}{2}z^{-1}} + \frac{B}{1+3z^{-1}}$$

$$A = \left.\frac{3\theta}{1+3\theta}\right|_{\theta=2} = \frac{6}{1+6} = \frac{6}{7}$$

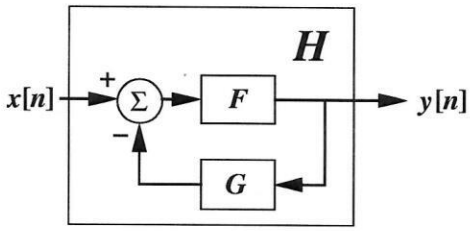
$$B = \left.\frac{3\theta}{1-\frac{1}{2}\theta}\right|_{\theta=-\frac{1}{3}} = \frac{3(-\frac{1}{3})}{1+\frac{1}{6}} = -\frac{1}{7/6} = -\frac{6}{7}$$

$$Y(z) = \underbrace{\frac{6/7}{1-\frac{1}{2}z^{-1}}}_{|z|>\frac{1}{2}} - \underbrace{\frac{6/7}{1+3z^{-1}}}_{|z|<3}$$

Table: $y[n] = \frac{6}{7}(\frac{1}{2})^n u[n] - (-\frac{6}{7})(-3)^n u[-n-1]$

$$y[n] = \tfrac{6}{7}(\tfrac{1}{2})^n u[n] + \tfrac{6}{7}(-3)^n u[-n-1]$$

2. **25/20 pts.** The causal, stable discrete-time LTI system H is a negative feedback connection of two causal, stable discrete-time LTI systems F and G as shown in the figure below:



The impulse response of system F is given by

$$f[n] = \left(\frac{1}{2}\right)^n u[n].$$

Table: $F(e^{j\omega}) = \dfrac{1}{1-\frac{1}{2}e^{-j\omega}}$

The impulse response of system G is given by

$$g[n] = \delta[n-1].$$

Table: $G(e^{j\omega}) = e^{-j\omega}$

(a) **13/10 pts.** Find the frequency response $H(e^{j\omega})$ of the overall system H.

$$H(e^{j\omega}) = \frac{F(e^{j\omega})}{1+F(e^{j\omega})G(e^{j\omega})} = \frac{\frac{1}{1-\frac{1}{2}e^{-j\omega}}}{1+\frac{e^{-j\omega}}{1-\frac{1}{2}e^{-j\omega}}} \cdot \underbrace{\frac{1-\frac{1}{2}e^{-j\omega}}{1-\frac{1}{2}e^{-j\omega}}}_{\text{one}}$$

$$= \frac{1}{1-\frac{1}{2}e^{-j\omega}+e^{-j\omega}} = \frac{1}{1+\frac{1}{2}e^{-j\omega}}$$

$$\boxed{H(e^{j\omega}) = \frac{1}{1+\frac{1}{2}e^{-j\omega}}}$$

Problem 2, cont...

(b) **12/10 pts.** Find the system output $y[n]$ when the input is given by

$$x[n] = \cos\left(\frac{\pi}{2}n\right).$$

→ The input signal is a cosine with frequency $\omega_0 = \pi/2$.

→ Table: $h[n] = \left(-\frac{1}{2}\right)^n u[n]$

→ Since $h[n]$ is real, we can apply the eigenfunction interpretation to find $y[n]$.

$$y[n] = \left|H(e^{j\pi/2})\right| \cos\left[\frac{\pi}{2}n + \arg H(e^{j\pi/2})\right]$$

$$e^{-j\pi/2} = -j$$

$$H(e^{j\pi/2}) = \frac{1}{1-\frac{1}{2}j} \cdot \underbrace{\frac{1+\frac{1}{2}j}{1+\frac{1}{2}j}}_{\text{one}} = \frac{1+\frac{1}{2}j}{1+\frac{1}{4}} = \frac{1+\frac{1}{2}j}{5/4} = \frac{4}{5} + j\frac{2}{5}$$

$$\left|H(e^{j\pi/2})\right| = \sqrt{\frac{16}{25} + \frac{4}{25}} = \sqrt{\frac{20}{25}} = \frac{\sqrt{20}}{5} = \frac{2\sqrt{5}}{5} = \frac{2}{\sqrt{5}} = 0.894427$$

$$\arg H(e^{j\pi/2}) = \arctan\left(\frac{2/5}{4/5}\right) = \arctan\frac{1}{2} = 0.463648$$

$$\boxed{y[n] = \frac{2}{\sqrt{5}} \cos\left[\frac{\pi}{2}n + 0.463648\right]}$$

3. **25/20 pts.** The input $x[n]$ and output $y[n]$ of a discrete-time LTI system H are related by

$$y[n] = x[n] + 2x[n-1] - 2x[n-2] - x[n-3].$$

(a) **3/2 pts.** Is the system H an FIR filter or an IIR filter? Justify your answer.

FIR. Because the I/O equation has no shifts of $y[n]$.

(b) **4/3 pts.** Find the impulse response $h[n]$.

Let $x[n] = \delta[n]$. Then $y[n] = h[n]$ and the I/O equation gives us

$$h[n] = \delta[n] + 2\delta[n-1] - 2\delta[n-2] - \delta[n-3]$$

→ Note that $h[n]$ has odd symmetry about the midpoint of the graph... it is a linear phase FIR filter.

(c) **4/3 pts.** Find the frequency response $H(e^{j\omega})$.

Table:

$$H(e^{j\omega}) = 1 + 2e^{-j\omega} - 2e^{-j2\omega} - e^{-j3\omega}$$

Problem 3, cont...

(d) **9/8 pts.** Find real-valued functions $A(\omega)$ and $\theta(\omega)$ such that $H(e^{j\omega}) = A(\omega)e^{j\theta(\omega)}$.

Highest power : $e^{-j3\omega}$. Half the highest power : $e^{-j\frac{3}{2}\omega}$

$$H(e^{j\omega}) = \left[e^{j\frac{3}{2}\omega} + 2e^{j\frac{1}{2}\omega} - 2e^{-j\frac{1}{2}\omega} - e^{-j\frac{3}{2}\omega}\right] e^{-j\frac{3}{2}\omega}$$

$$= 2j\left[\frac{e^{j\frac{3}{2}\omega} - e^{-j\frac{3}{2}\omega}}{2j} + 2\,\frac{e^{j\frac{1}{2}\omega} - e^{-j\frac{1}{2}\omega}}{2j}\right] e^{-j\frac{3}{2}\omega}$$

$$= 2\left[\sin\tfrac{3}{2}\omega + 2\sin\tfrac{1}{2}\omega\right] \underbrace{e^{j\pi/2}}_{j}\, e^{-j\frac{3}{2}\omega}$$

$$= \left[2\sin\tfrac{3}{2}\omega + 4\sin\tfrac{1}{2}\omega\right] e^{j\left(\frac{\pi}{2} - \frac{3}{2}\omega\right)}$$

$$A(\omega) = 2\sin\tfrac{3}{2}\omega + 4\sin\tfrac{1}{2}\omega$$

$$\theta(\omega) = \tfrac{\pi}{2} - \tfrac{3}{2}\omega$$

(e) **5/4 pts.** Find the system group delay $\tau(\omega)$.

$$\tau(\omega) = -\frac{d}{d\theta}\theta(\omega) = -\frac{d}{d\omega}\left\{\tfrac{\pi}{2} - \tfrac{3}{2}\omega\right\} = \tfrac{3}{2}$$

$$\tau(\omega) = \frac{3}{2}$$

4. **25/20 pts.** The input $x[n]$ and output $y[n]$ of a causal discrete-time LTI system H are related by

$$y[n] - \frac{1}{5}y[n-1] = x[n] - \frac{1}{2}x[n-1].$$

The input signal for $n \geq 0$ is given by

$$x[n] = \left(-\frac{1}{2}\right)^n u[n], \qquad n \geq 0.$$

Table: $\mathcal{X}_u(z) = \dfrac{1}{1+\frac{1}{2}z^{-1}}$

Initial conditions on the output signal $y[n]$ and input signal $x[n]$ are given as

$$y[-1] = \frac{1}{7} \qquad \text{and} \qquad x[-1] = -2.$$

Use the unilateral z-transform to find $y[n]$ for $n \geq 0$.

$\mathcal{UZ}$: $\mathcal{Y}_u(z) - \frac{1}{5}\{z^{-1}\mathcal{Y}_u(z) + y[-1]\} = \mathcal{X}_u(z) - \frac{1}{2}\{z^{-1}\mathcal{X}_u(z) + x[-1]\}$

$\mathcal{Y}_u(z) - \frac{1}{5}z^{-1}\mathcal{Y}_u(z) - \frac{1}{5}\left(\frac{1}{7}\right) = \mathcal{X}_u(z) - \frac{1}{2}z^{-1}\mathcal{X}_u(z) - \frac{1}{2}(-2)$

$\mathcal{Y}_u(z)\left[1 - \frac{1}{5}z^{-1}\right] - \frac{1}{35} = \mathcal{X}_u(z)\left[1 - \frac{1}{2}z^{-1}\right] + 1$

$\mathcal{Y}_u(z)\left[1 - \frac{1}{5}z^{-1}\right] = \frac{1}{1+\frac{1}{2}z^{-1}}\left[1 - \frac{1}{2}z^{-1}\right] + 1 + \frac{1}{35}$

$\mathcal{Y}_u(z) = \frac{1-\frac{1}{2}z^{-1}}{(1+\frac{1}{2}z^{-1})(1-\frac{1}{5}z^{-1})} + \frac{36/35}{1-\frac{1}{5}z^{-1}}$

PFE: $\dfrac{1-\frac{1}{2}\theta}{(1+\frac{1}{2}\theta)(1-\frac{1}{5}\theta)} = \dfrac{A}{1+\frac{1}{2}\theta} + \dfrac{B}{1-\frac{1}{5}\theta}$

$A = \left.\dfrac{1-\frac{1}{2}\theta}{1-\frac{1}{5}\theta}\right|_{\theta=-2} = \dfrac{1+1}{1+2/5} = \dfrac{2}{7/5} = \dfrac{10}{7}$

$B = \left.\dfrac{1-\frac{1}{2}\theta}{1+\frac{1}{2}\theta}\right|_{\theta=5} = \dfrac{1-5/2}{1+5/2} = \dfrac{-3/2}{7/2} = \dfrac{-3}{7}$

More Workspace for Problem 4...

$$Y_u(z) = \frac{10/7}{1+\frac{1}{2}z^{-1}} - \frac{3/7}{1-\frac{1}{5}z^{-1}} + \frac{36/35}{1-\frac{1}{5}z^{-1}}$$

$$= \frac{10/7}{1+\frac{1}{2}z^{-1}} + \frac{\frac{36-15}{35}}{1-\frac{1}{5}z^{-1}}$$

$$= \frac{10/7}{1+\frac{1}{2}z^{-1}} + \frac{21/35}{1-\frac{1}{5}z^{-1}} \quad \leftarrow \quad \frac{21}{35} = \frac{3 \cdot 7}{5 \cdot 7} = 3/5$$

Table:

$$y[n] = \frac{10}{7}\left(-\frac{1}{2}\right)^n u[n] + \frac{3}{5}\left(\frac{1}{5}\right)^n u[n] \quad \text{for } n \geq 0$$

5. **25/20 pts.** A discrete-time LTI system H has impulse response $h[n]$ given by

$$h[n] = 5\left(-\frac{1}{4}\right)^n u[n] + 4\,(-2)^n\,u[-n-1].$$

(a) **10/8 pts.** Find the transfer function $H(z)$, specify the ROC, and give a pole-zero plot.

$h[n] = 5\left(-\frac{1}{4}\right)^n u[n] - \{-4\,(-2)^n u[-n-1]\}$

Table: $H(z) = \underbrace{\frac{5}{1+\frac{1}{4}z^{-1}}}_{|z|>\frac{1}{4}} - \underbrace{\frac{4}{1+2z^{-1}}}_{|z|<2} = \frac{5(1+2z^{-1}) - 4(1+\frac{1}{4}z^{-1})}{(1+\frac{1}{4}z^{-1})(1+2z^{-1})}$

$= \frac{5+10z^{-1}-4-z^{-1}}{(1+\frac{1}{4}z^{-1})(1+2z^{-1})} = \boxed{\frac{1+9z^{-1}}{(1+\frac{1}{4}z^{-1})(1+2z^{-1})}, \quad \frac{1}{4}<|z|<2}$

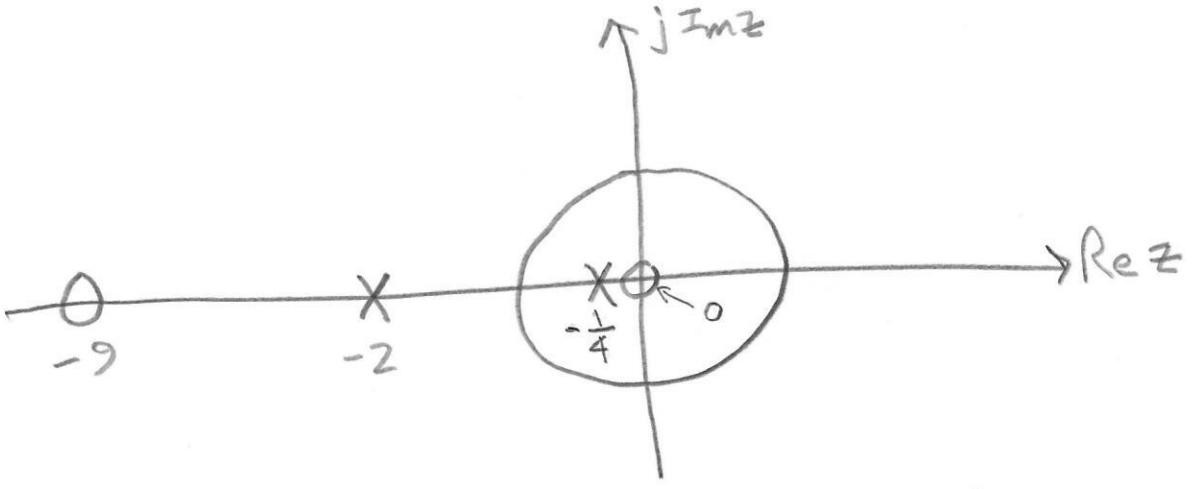
$H(z)\cdot\frac{z^2}{z^2} = \frac{z(z+9)}{(z+\frac{1}{4})(z+2)}$

zeros: $z = 0, -9$

poles: $z = -\frac{1}{4}, -2$

$\lim_{z\to\infty} H(z) = \lim_{z\to\infty} \frac{z^2}{z^2} = 1$

→ no additional poles or zeros @ $z=\infty$.



Problem 5, cont...

(b) **5/4 pts.** Find the difference equation (I/O equation) that relates the system input signal $x[n]$ to the output signal $y[n]$.

$$H(z) = \frac{1+9z^{-1}}{(1+\frac{1}{4}z^{-1})(1+2z^{-1})} = \frac{1+9z^{-1}}{1+\frac{1}{4}z^{-1}+2z^{-1}+\frac{1}{2}z^{-2}}$$

$$= \frac{1+9z^{-1}}{1+9/4\,z^{-1}+1/2\,z^{-2}} = \frac{Y(z)}{X(z)}$$

$$[1+9z^{-1}]X(z) = [1+9/4z^{-1}+1/2z^{-2}]Y(z)$$

$$X(z)+9z^{-1}X(z) = Y(z)+9/4z^{-1}Y(z)+\frac{1}{2}z^{-2}Y(z)$$

z^{-1}: $y[n]+9/4\,y[n-1]+\frac{1}{2}y[n-2] = x[n]+9x[n-1]$

Problem 5, cont...

(c) **10/8 pts.** Find the system output $y[n]$ when the input is given by

Table: $X(z) = \dfrac{15/2}{1-\frac{1}{2}z^{-1}}$, $|z|>\frac{1}{2}$ $x[n] = \frac{15}{2}\left(\frac{1}{2}\right)^n u[n]$.

$$Y(z) = X(z)H(z) = \frac{\frac{15}{2}(1+9z^{-1})}{(1+\frac{1}{4}z^{-1})(1+2z^{-1})(1-\frac{1}{2}z^{-1})}, \quad \frac{1}{2}<|z|<2$$

$$= \frac{A}{1+\frac{1}{4}z^{-1}} + \frac{B}{1+2z^{-1}} + \frac{C}{1-\frac{1}{2}z^{-1}}$$

$$A = \left.\frac{15/2(1+9\theta)}{(1+2\theta)(1-\frac{1}{2}\theta)}\right|_{\theta=-4} = \frac{\frac{15}{2}(1-36)}{(1-8)(1+2)} = \frac{\frac{15}{2}(-35)}{(-7)(3)} = \frac{25}{2}$$

$$B = \left.\frac{\frac{15}{2}(1+9\theta)}{(1+\frac{1}{4}\theta)(1-\frac{1}{2}\theta)}\right|_{\theta=-\frac{1}{2}} = \frac{\frac{15}{2}(1-9/2)}{(1-\frac{1}{8})(1+\frac{1}{4})} = \frac{\frac{15}{2}(-\frac{7}{2})}{(7/8)(5/4)} = -24$$

$$C = \left.\frac{\frac{15}{2}(1+9\theta)}{(1+\frac{1}{4}\theta)(1+2\theta)}\right|_{\theta=2} = \frac{\frac{15}{2}(1+18)}{(1+\frac{1}{2})(1+4)} = \frac{\frac{15}{2}(19)}{(\frac{3}{2})5} = 19$$

$$Y(z) = \underbrace{\frac{25/2}{1+\frac{1}{4}z^{-1}}}_{|z|>\frac{1}{4}} - \underbrace{\frac{24}{1+2z^{-1}}}_{|z|<2} + \underbrace{\frac{19}{1-\frac{1}{2}z^{-1}}}_{|z|>\frac{1}{2}}$$

Table: $y[n] = \frac{25}{2}\left(-\frac{1}{4}\right)^n u[n] + 24(-2)^n u[-n-1] + 19\left(\frac{1}{2}\right)^n u[n]$