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Section:

Laboratory Exercise 4

LINEAR, TIME-INVARIANT DISCRETE-TIME SYSTEMS: FREQUENCY-DOMAIN REPRESENTATIONS

4.1   TRANSFER FUNCTION AND FREQUENCY RESPONSE

**Project 4.1   Transfer Function Analysis**

Answers:

Q4.1   The modified Program P3_1 to compute and plot the magnitude and phase spectra of a moving average filter of Eq. (2.13) for $0 \leq \omega \leq 2\pi$ is shown below:

```
% Program Q4_1
% Frequency response of the causal M-point averager of Eq. (2.13)
clear;
% User specifies filter length
M = input('Enter the filter length M: ');
% Compute the frequency samples of the DTFT
w = 0:2*pi/1023:2*pi;
num = (1/M)*ones(1,M);
den = [1];
% Compute and plot the DTFT
h = freqz(num, den, w);
subplot(2,1,1)
plot(w/pi,abs(h));grid
title('Magnitude Spectrum |H(e^{j\omega})|')
xlabel('\omega /\pi');
ylabel('Amplitude');
subplot(2,1,2)
plot(w/pi,angle(h));grid
title('Phase Spectrum arg[H(e^{j\omega})]')
xlabel('\omega /\pi');
ylabel('Phase in radians');
```

This program was run for the following three different values of M and the plots of the corresponding frequency responses are shown below:
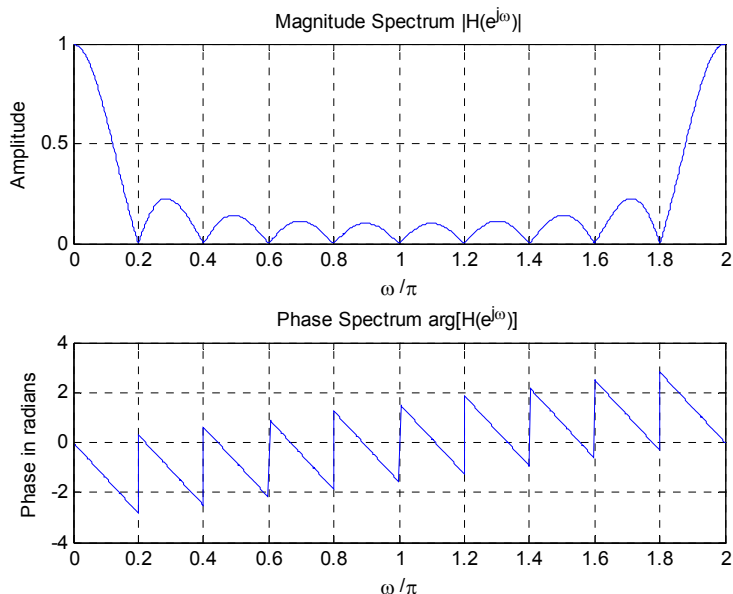
$$M=3$$

Magnitude Spectrum $|H(e^{j\omega})|$

Phase Spectrum $\arg[H(e^{j\omega})]$

$$M=7$$

Magnitude Spectrum $|H(e^{j\omega})|$
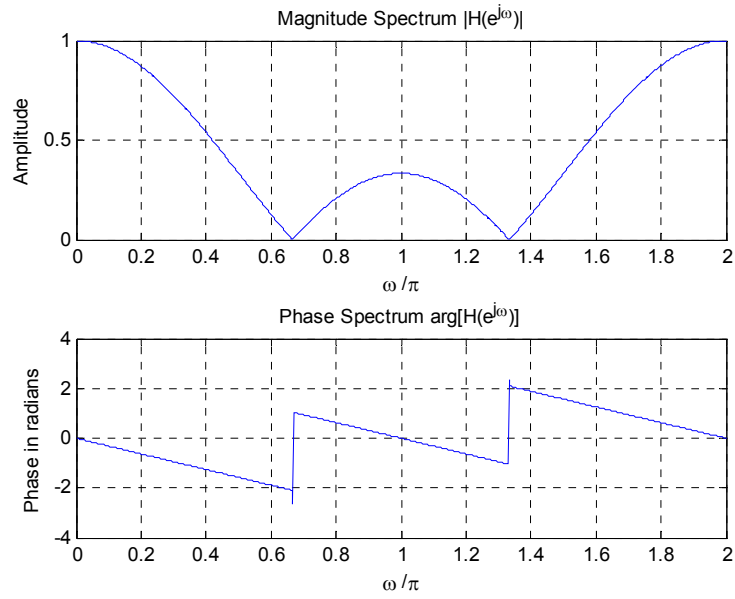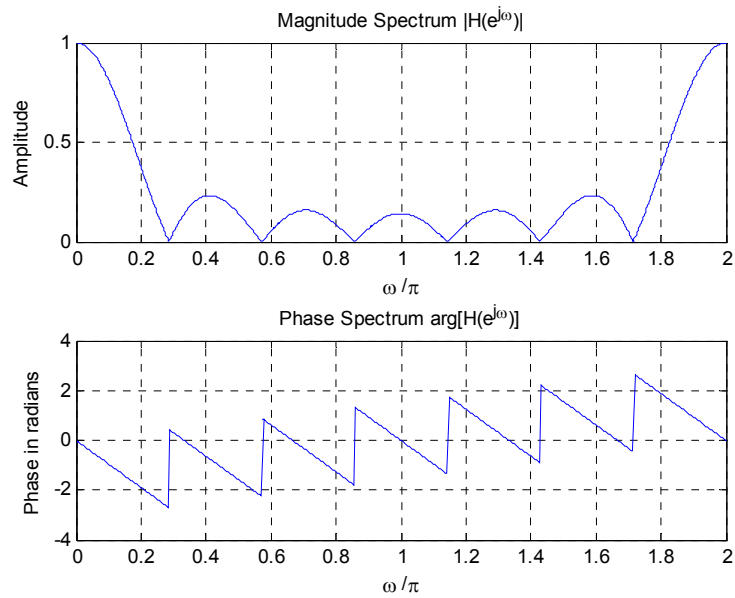
Phase Spectrum $\arg[H(e^{j\omega})]$

M=10

The types of symmetries exhibited by the magnitude and phase spectra are due to - The impulse response is real. Therefore, the frequency response is periodically conjugate symmetric, the magnitude response is periodically even symmetric, and the phase response is periodically odd symmetric.

The type of filter represented by the moving average filter is - This is a lowpass filter.

The results of Question Q2.1 can now be explained as follows – It is a lowpass filter. The input was a sum of two sinusoids, one high frequency and one low frequency. The particular results depend on the filter length M, but the general result is that the higher frequency sinusoidal input component is attenuated more than the lower frequency sinusoidal input component.

Q4.2   The plot of the frequency response of the causal LTI discrete-time system of Question Q4.2 obtained using the modified program is given below:

Magnitude Spectrum |H(e^jω)|

Phase Spectrum arg[H(e^jω)]
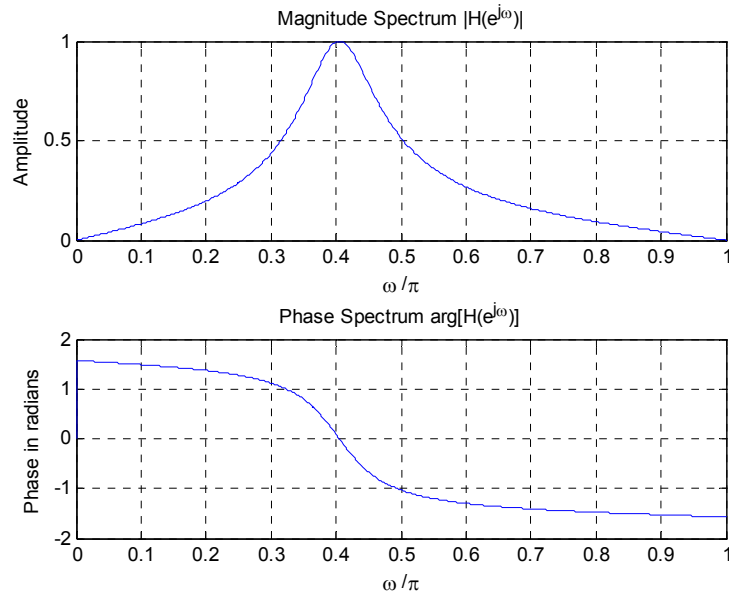
The type of filter represented by this transfer function is - Bandpass

Q4.3   The plot of the frequency response of the causal LTI discrete-time system of Question Q4.3 obtained using the modified program is given below:

Magnitude Spectrum |H(e^jω)|

Phase Spectrum arg[H(e^jω)]
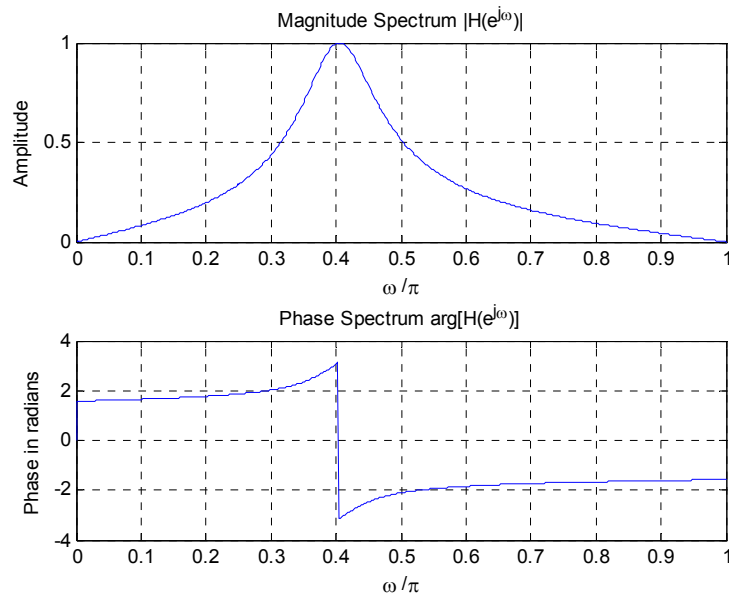
The type of filter represented by this transfer function is - Bandpass

The difference between the two filters of Questions 4.2 and 4.3 is - The magnitude responses are the same. The phase responses might look very different to you at first,

but they are actually similar. The phase response of the second filter exhibits a branch cut of the arctangent function at normalized frequency 0.4, which is right in the middle of the passband. However, the unwrapped phase would not show this discontinuity. Both filters have an approximately linear phase in the passband and their group delays are approximately the negatives of one another. However, the two filters have different poles. The poles of the first filter are INSIDE the unit circle, whereas those of the second filer are OUTSIDE the unit circle. Thus, in a causal implementation, the first filter would be BIBO STABLE, whereas the second filter would be UNSTABLE. Therefore, in most applications the second filter would be preferable.

I shall choose the filter of Question Q4.__36__ for the following reason - It can be both causal and BIBO stable, whereas the filter of Q4.37 cannot be both because the two poles are both outside of the unit circle.

Q4.4   The group delay of the filter specified in Question Q4.4 and obtained using the function `grpdelay` is shown below:


Group Delay of $H(e^{j\omega})$
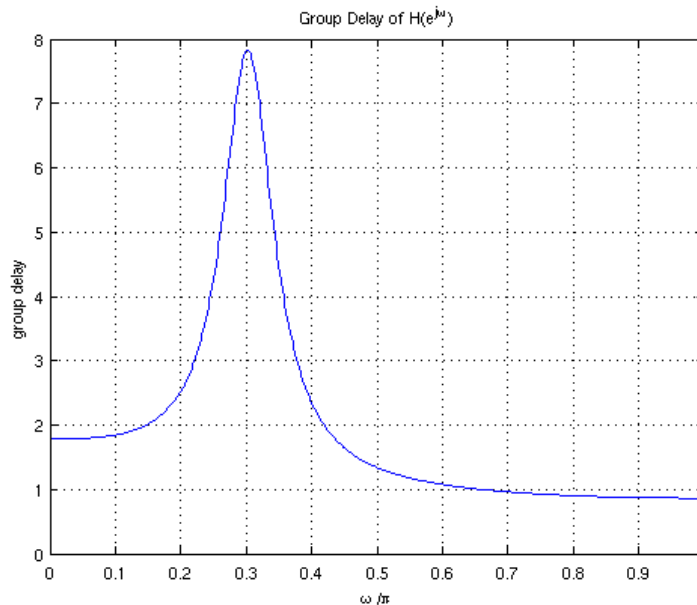
From this plot we make the following observations: Usually, it is desirable for a filer to have an approximately linear phase in the passband, which is the same thing as an approximately constant group delay in the passband. This filter is notch filter; it is a bandstop filter with a narrow stopband centered at a normalized frequency just below 0.3. From the graph above, we see that the group delay is approximately constant over much of the passband.

Q4.5   The plots of the first 100 samples of the impulse responses of the two filters of Questions 4.2 and 4.3 obtained using the program developed in Question Q3.50 are shown below:

5

First 100 samples of h[n] for (4.36)

First 100 samples of h[n] for (4.37)
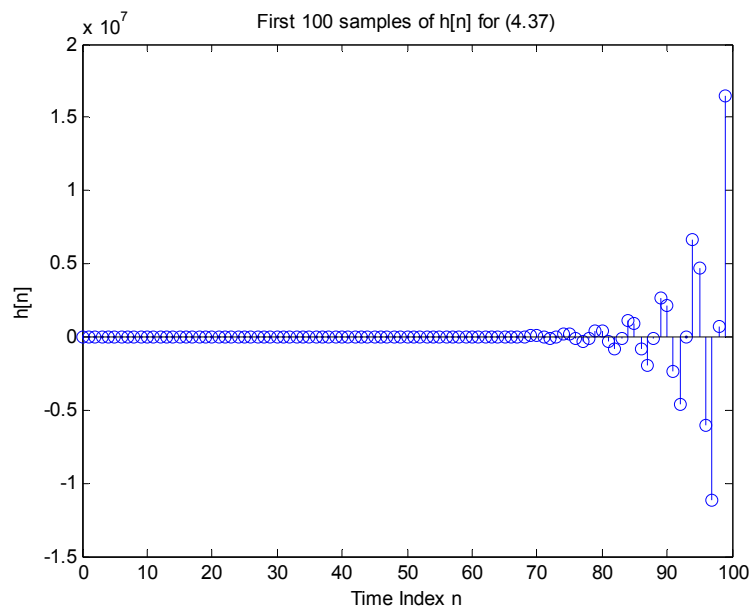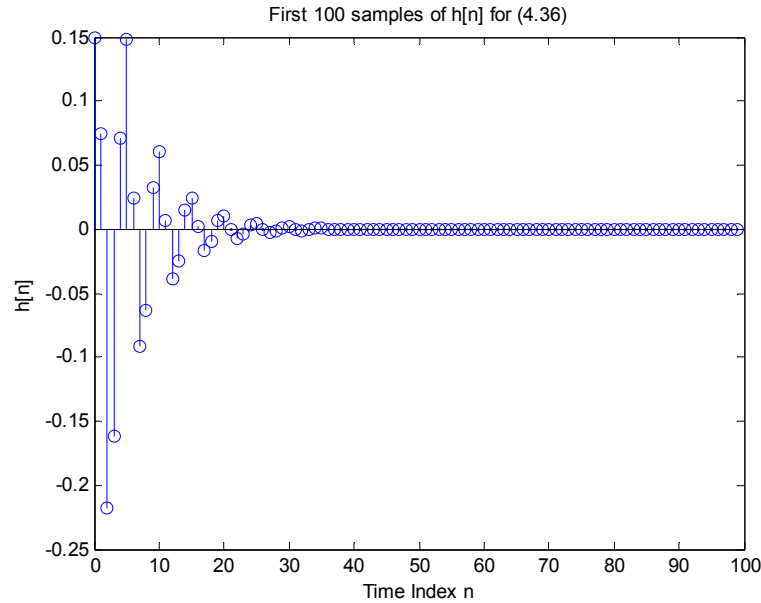
From these plots we make the following observations: These plots generated by impz give the impulse response for a causal realization of H(z). As we observed in Q4.3, the causal filter with H(z) given in (4.36) is BIBO stable, implying that h[n] is absolutely summable, and we see alternation and exponential decay in the impulse response. On the other hand, the causal filer with H(z) given in (4.37) has poles outside the unit circle and is unstable. Not surprisingly, corresponding h[n] shown above displays exponential growth with n.

Q4.6 The pole-zero plots of the two filters of Questions 4.2 and 4.3 developed using `zplane` are shown below:
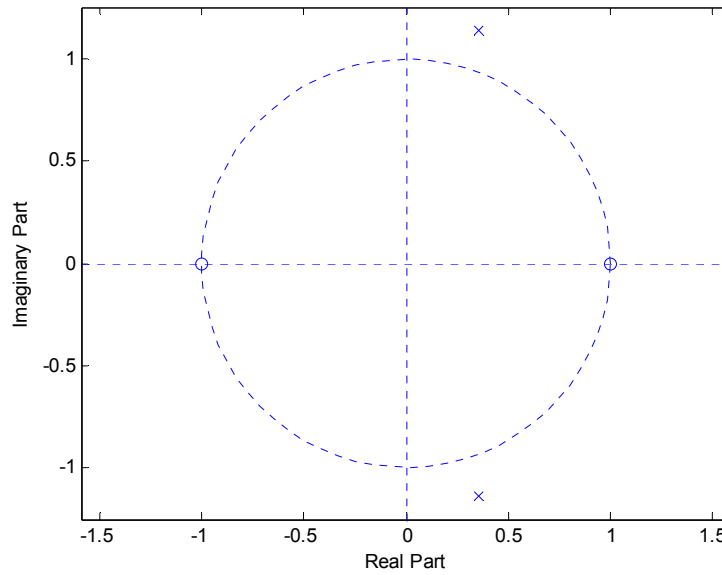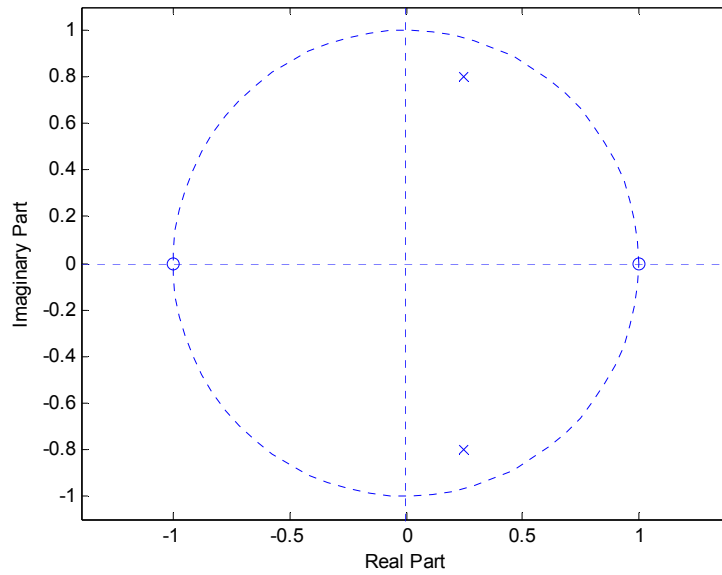
From these plots we make the following observations: As we have said repeatedly by this time, the upper figure shows that the filter in (4.36) has poles inside the unit circle and therefore it's causal realization is stable. The lower figure shows that the filter in (4.37) has poles outside the unit circle; its causal realization is unstable.

**4.2   TYPES OF TRANSFER FUNCTIONS**

Project 4.2   Filters

A copy of Program P4_1 is given below:

```
% Program P4_1
% Impulse Response of Truncated Ideal Lowpass Filter
clf;
fc = 0.25;
n = [-6.5:1:6.5];
y = 2*fc*sinc(2*fc*n);k = n+6.5;
stem(k,y);title('N = 13');axis([0 13 -0.2 0.6]);
xlabel('Time index n');ylabel('Amplitude');grid;
```

Answers:

Q4.7   The plot of the impulse response of the approximation to the ideal lowpass filter obtained using Program P4_1 is shown below:
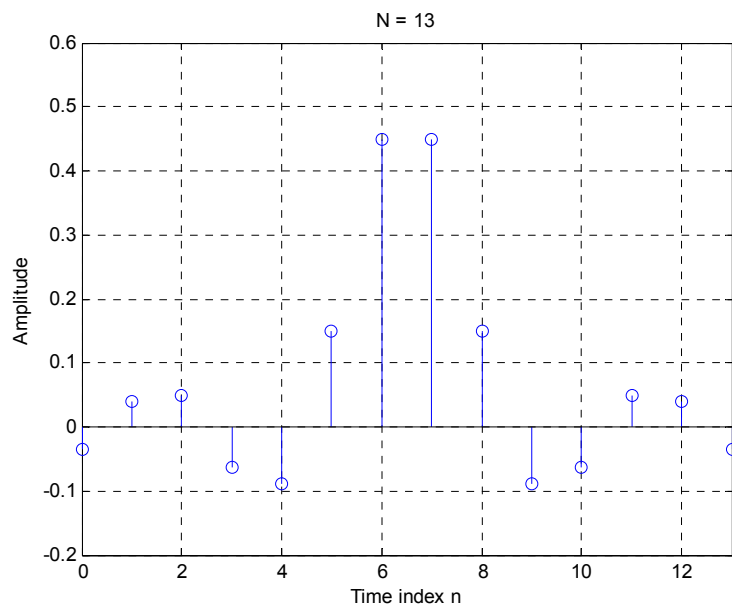
The length of the FIR lowpass filter is - 14
The statement in Program P4_1 determining the filter length is -
        n = [-6.5:1:6.5];

The parameter controlling the cutoff frequency is - `fc = 0.25;`

Q4.8   The required modifications to Program P4_1 to compute and plot the impulse response of the FIR lowpass filter of Project 4.2 with a length of 20 and a cutoff frequency of $\omega_c = 0.45$ are as indicated below:

```
% Program Q4_8
% Impulse Response of Truncated Ideal Lowpass Filter
clf;
```
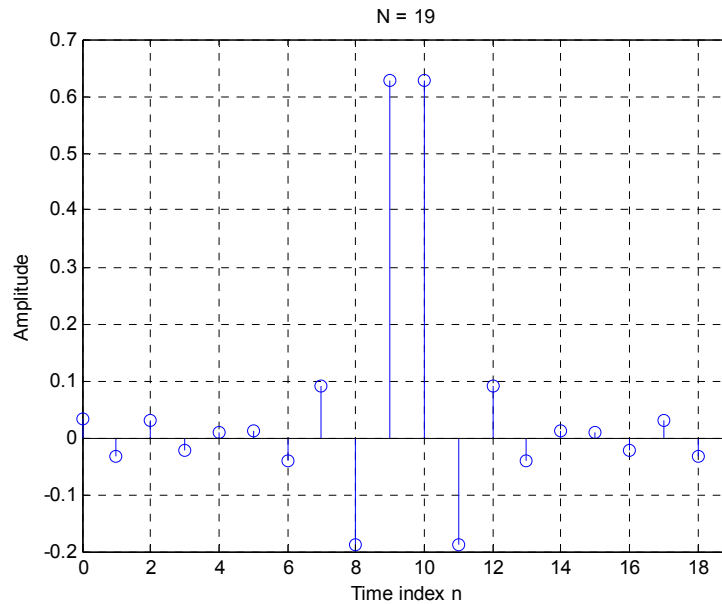
8

```
fc = 0.45;
n = [-9.5:1:9.5];
y = 2*fc*sinc(2*fc*n);k = n+9.5;
stem(k,y);title('N = 20');axis([0 19 -0.2 0.7]);
xlabel('Time index n');ylabel('Amplitude');grid;
```

The plot generated by running the modified program is given below:

Q4.9   The required modifications to Program P4_1 to compute and plot the impulse response of the FIR lowpass filter of Project 4.2 with a length of 15 and a cutoff frequency of $\omega_c = 0.65$ are as indicated below:

```
% Program Q4_9
% Impulse Response of Truncated Ideal Lowpass Filter
clf;
fc = 0.65;
n = [-7.0:1:7.0];
y = 2*fc*sinc(2*fc*n);k = n+7.0;
stem(k,y);title('N = 14');axis([0 14 -0.4 1.4]);
xlabel('Time index n');ylabel('Amplitude');grid;
```

The plot generated by running the modified program is given below:

N = 14
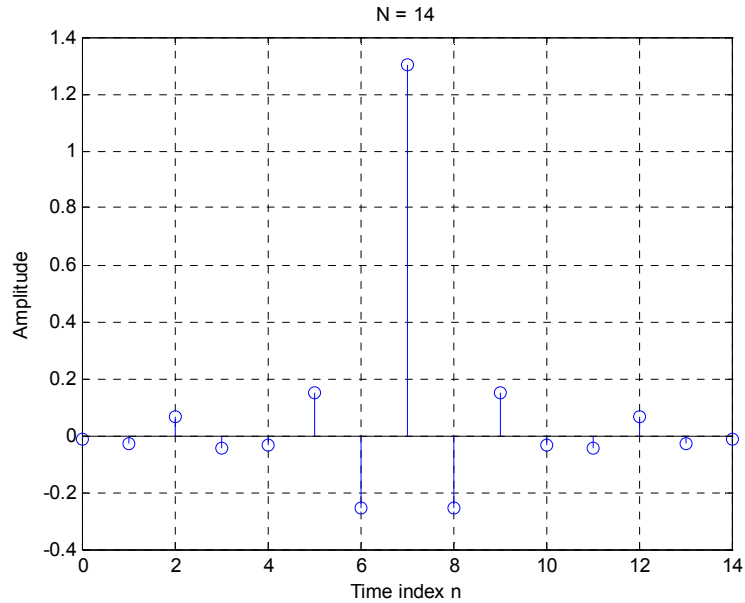
Q4.10   The MATLAB program to compute and plot the amplitude response of the FIR lowpass filter of Project 4.2 is given below:

```
% Program Q4_10
% Compute and plot the amplitude response
% of Truncated Ideal Lowpass Filter
clear;
% Get "N" from the user command line
N = input('Enter the filter time shift N: ');
% compute the magnitude spectrum
No2 = N/2;
fc = 0.25;
n = [-No2:1:No2];
y = 2*fc*sinc(2*fc*n);
w = 0:pi/511:pi;
h = freqz(y, [1], w);
plot(w/pi,abs(h));
grid;
title(strcat('|H(e^{j\omega})|, N=',num2str(N)));
xlabel('\omega /\pi');
ylabel('Amplitude');
```

Plots of the amplitude response of the lowpass filter for several values of N are shown below:

10

|H(e^{j\omega})|, N=13

|H(e^{j\omega})|, N=14

11

$|H(e^{j\omega})|$, N=19


$|H(e^{j\omega})|$, N=99
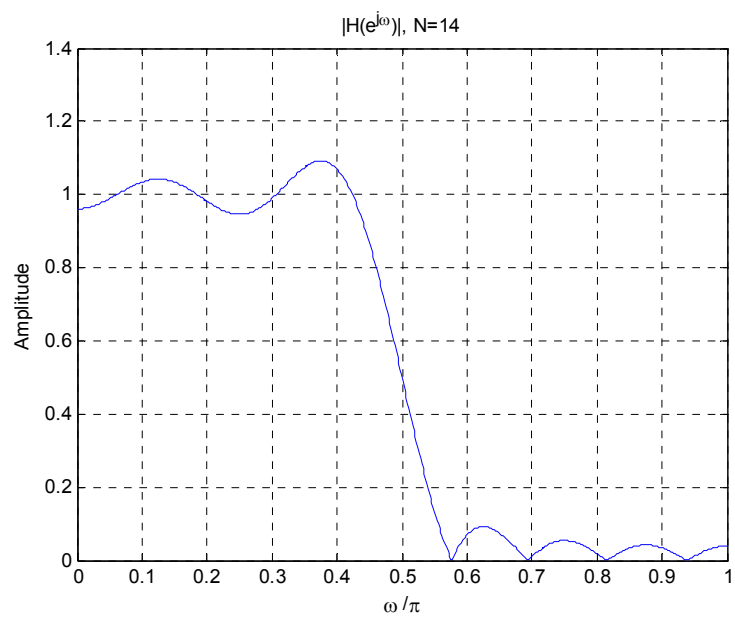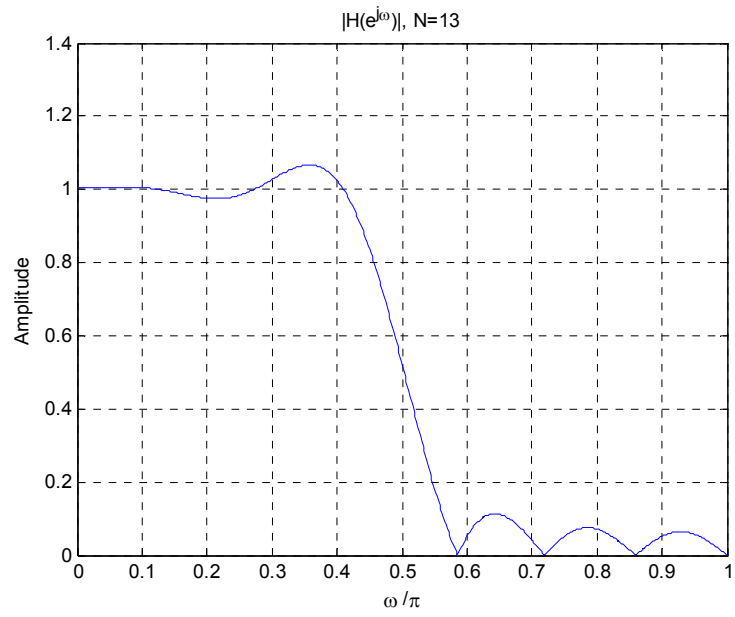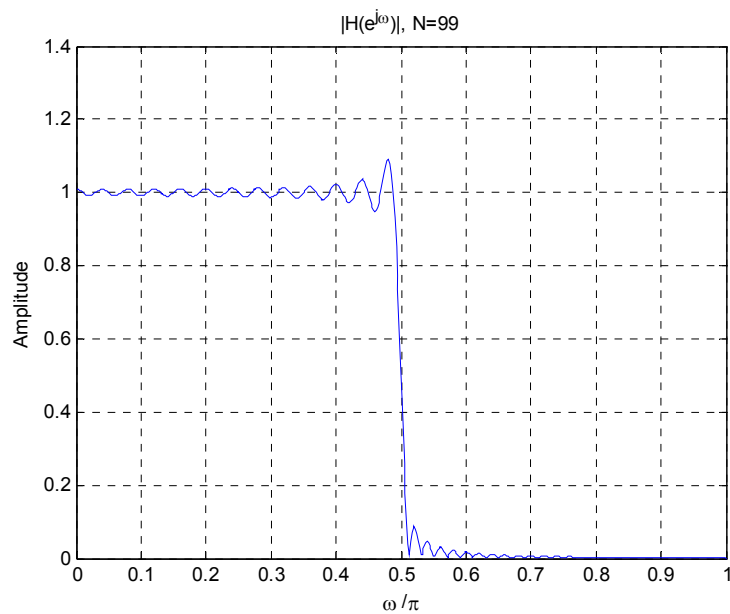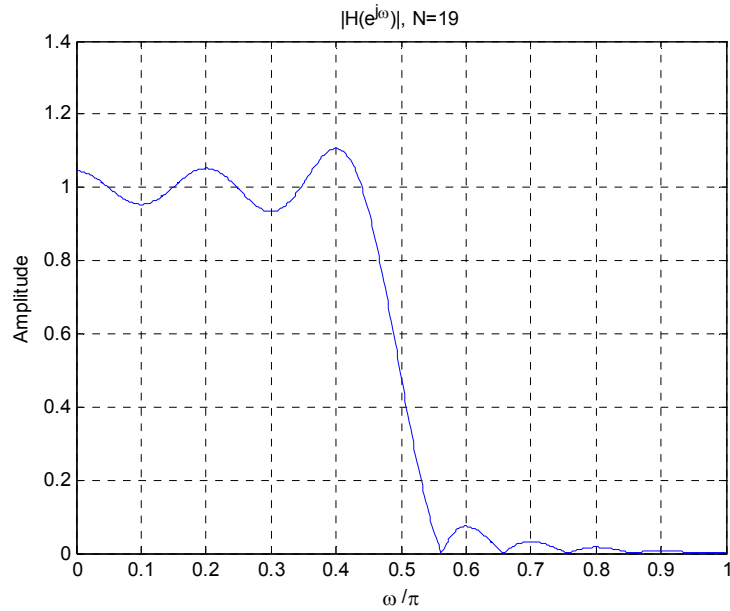
From these plots we can make the following observations – As the filter length increases, the transition from passband to stopband becomes steeper; i.e., the transition band becomes more narrow. We also see the Gibb's phenomena: as the filter order increases, the amplitude response converges towards the ideal lowpass characteristic. However, the convergence is only in weak sense – pointwise convergence. The peak ripple at the transition band edges does not decrease despite the increases in order.

A copy of Program P4_2 is given below:

```
% Program P4_2
% Gain Response of a Moving Average Lowpass Filter
clf;
M = 2;
num = ones(1,M)/M;
[g,w] = gain(num,1);
plot(w/pi,g);grid
axis([0 1 -50 0.5])
xlabel('\omega /\pi');ylabel('Gain in dB');
title(['M = ', num2str(M)])
```

Answers:

Q4.11   A plot of the gain response of a length-2 moving average filter obtained using Program P4_2 is shown below:
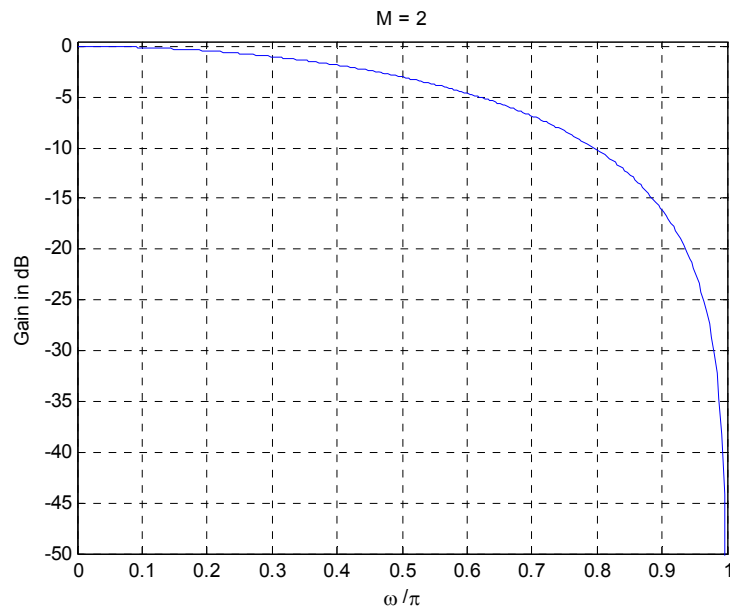
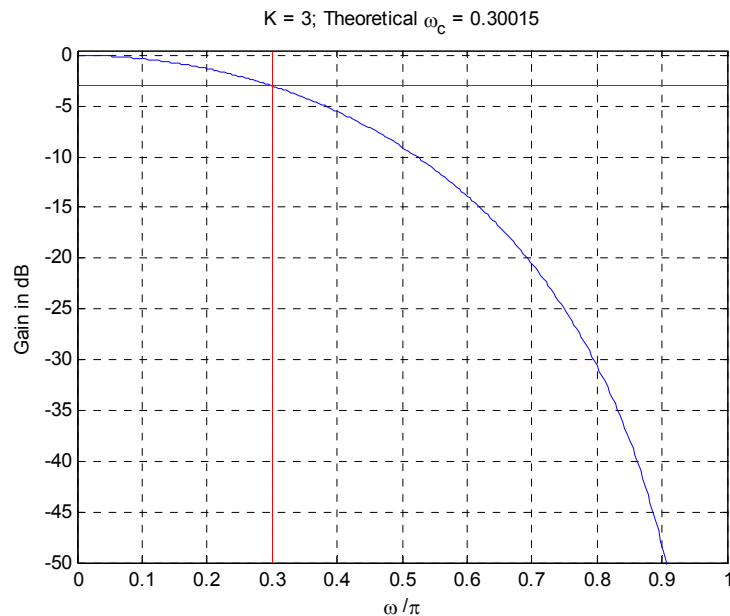From the plot it can be seen that the 3-dB cutoff frequency is at – $\pi/2$ rad/sample.

Q4.12   The required modifications to Program P4_2 to compute and plot the gain response of a cascade of K length-2 moving average filters are given below:

13

```
% Program Q4_12
% Gain Response of a cascade connection of K
% two-point Moving Average Lowpass Filters
clear;
K = input('Enter the number of sections K: ');
Hz = [1];
% find the numerator for H(z) = cascade of K sections
for i=1:K;
  Hz = conv(Hz,[1 1]);
end;
Hz = (0.5)^K * Hz;
% Convert numerator to dB
[g,w] = gain(Hz,1);
% make a horizontal line on the plot at -3 dB
ThreedB = -3*ones(1,length(g));
% make a vertical line on the plot at the
%  theoretical 3dB frequency
t1 = 2*acos((0.5)^(1/(2*K)))*ones(1,512)/pi;
t2 = -50:50.5/511:0.5;
plot(w/pi,g,w/pi,ThreedB,t1,t2);grid;
axis([0 1 -50 0.5])
xlabel('\omega /\pi');ylabel('Gain in dB');
title(['K = ',num2str(K),'; Theoretical \omega_{c} = ',num2str(t1(1))]);
```

The plot of the gain response for a cascade of 3 sections obtained using the modified program is shown below:

From the plot it can be seen that the 3-dB cutoff frequency of the cascade is at -0.30015π

Q4.13 The required modifications to Program P4_2 to compute and plot the gain response of the highpass filter of Eq. (4.42) are given below:

14

```
% Program Q4_13
% Gain Response of Highpass Filter (4.42)
clear;
M = input('Enter the filter length M: ');
n = 0:M-1;
num = (-1).^n .* ones(1,M)/M;
[g,w] = gain(num,1);
plot(w/pi,g);grid;
axis([0 1 -50 0.5]);
xlabel('\omega /\pi');ylabel('Gain in dB');
title(['M = ', num2str(M)]);
```

The plot of the gain response for M = 5 obtained using the modified program is shown below:
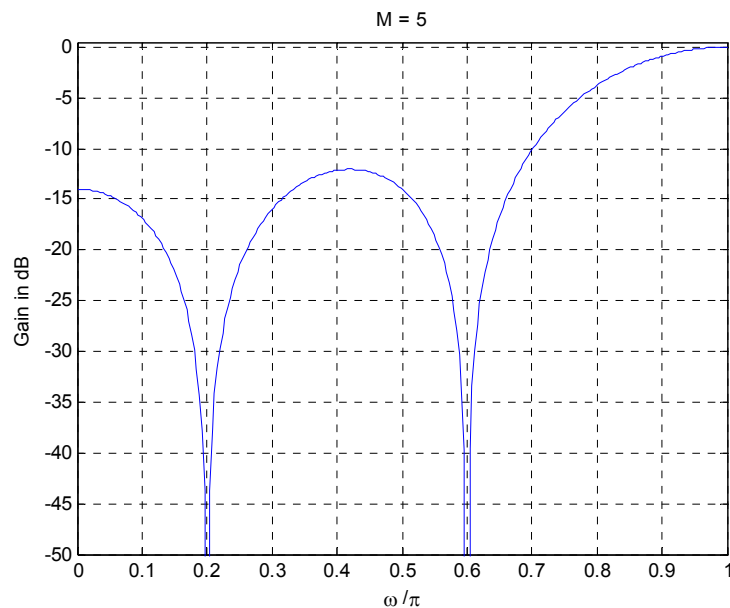
From the plot we can see that the 3-dB cutoff frequency is at – approximately 0.8196π
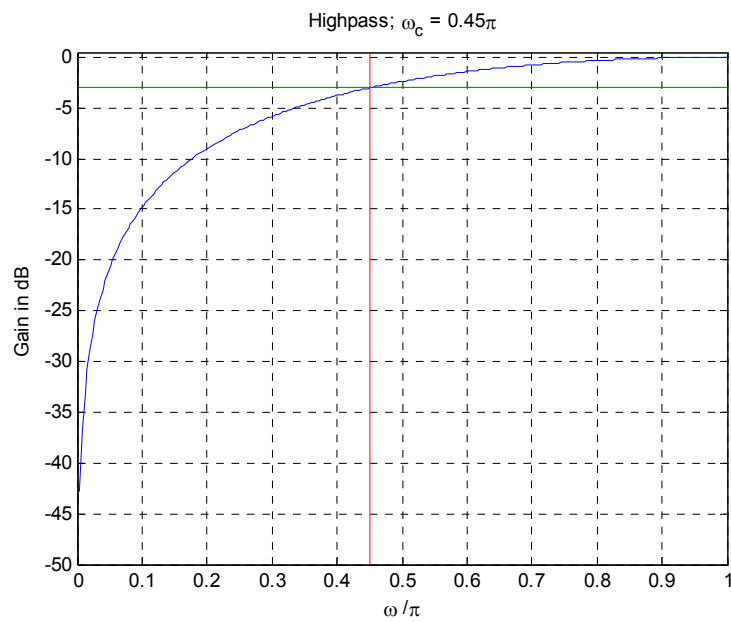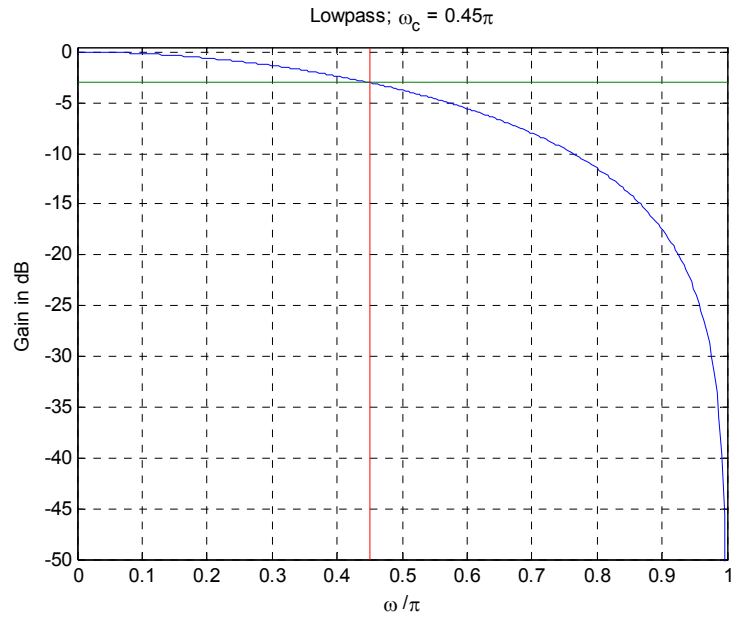
Q4.14   From Eq. (4.16) for a 3-dB cutoff frequency ω_{C} at 0.45π we obtain $\alpha = 0.078702$

Substituting this value of α in Eqs. (4.15) and (4.17) we arrive at the transfer function of the first-order IIR lowpass and highpass filters, respectively, given by

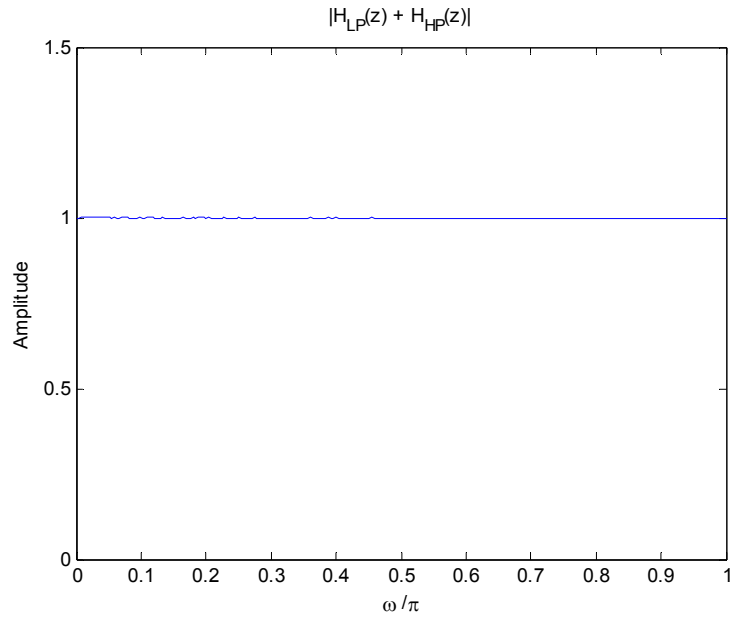$$H_{\text{LP}}(z) \approx \frac{0.46065(1+z^{-1})}{1-0.07870z^{-1}}$$

$$H_{\text{LP}}(z) \approx \frac{0.53935(1-z^{-1})}{1-0.07870z^{-1}}$$

The plots of their gain responses obtained using MATLAB are shown below:

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Lowpass; $\omega_c = 0.45\pi$

Highpass; $\omega_c = 0.45\pi$

From these plots we observe that the designed filters __DO__ meet the specifications.

A plot of the magnitude response of the sum $H_{LP}(z) + H_{HP}(z)$ obtained using MATLAB is given below:

16

$|H_{LP}(z) + H_{HP}(z)|$

From this plot we observe that the two filters are – Allpass Complementary

A plot of the sum of the square-magnitude responses of $H_{LP}(z)$ and $H_{HP}(z)$ obtained using MATLAB is given below:

$|H_{LP}(z)|^2 + |H_{HP}(z)|^2$
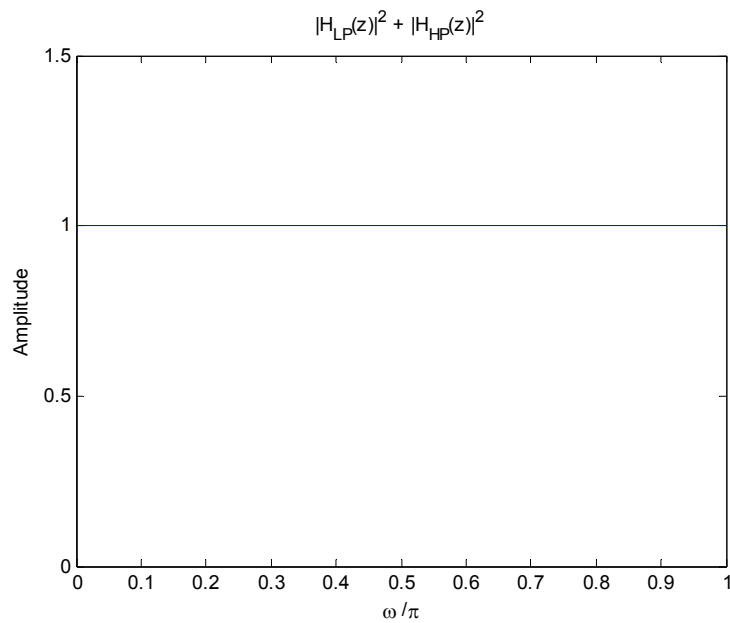
From this plot we observe that the two filters are – Power Complementary

Q4.15 From Eq. (4.24), we get substituting $K = 10$, $B = 1.8660660$

Substituting this value of B and $\omega_c = 0.3\pi$ in Eq. (4.23) we obtain $\alpha = $ -0.3107925

Using this value of α in Eq. (4.22) we arrive at the transfer function of the cascade of 10 IIR lowpass filters as
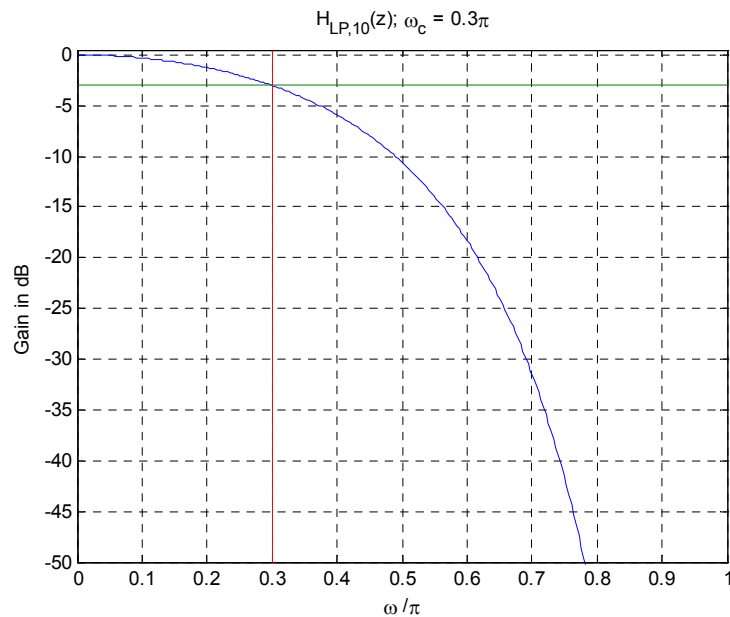
$$H_{LP,10}(z) = \left(\frac{1-\alpha}{2} \cdot \frac{1+z^{-1}}{1-\alpha z^{-1}} \right)^{10} \approx \frac{23.61566 \times 10^{-6} \left(1+z^{-1} \right)^{10}}{\left(1+0.31079 z^{-1} \right)^{10}}$$
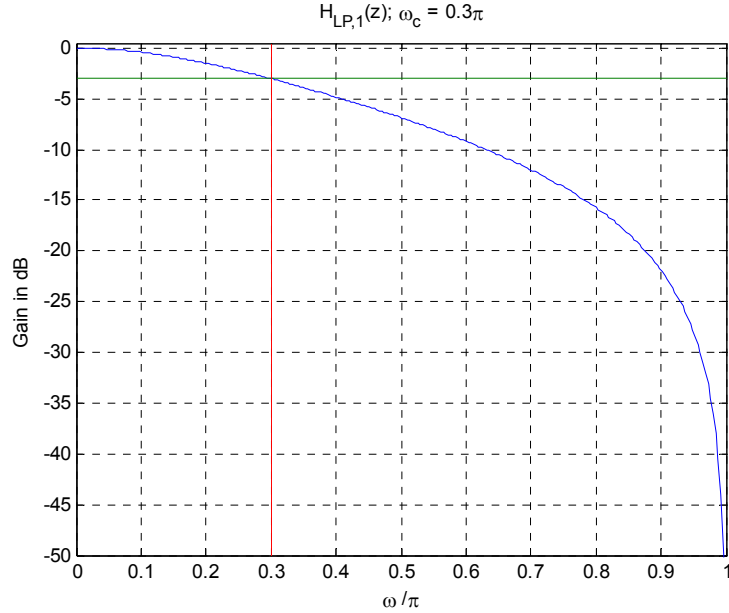
Substituting $\omega_c = 0.3\pi$ in Eq. (4.16) we obtain $\alpha = 0.3249197$

Using this value of α in Eq. (4.15) we arrive at the transfer function of a first-order IIR lowpass filter

$$H_{LP,1}(z) = \frac{1-\alpha}{2} \cdot \frac{1+z^{-1}}{1-\alpha z^{-1}} \approx \frac{0.33754(1+z^{-1})}{1-0.32492 z^{-1}}$$

The gain responses of $H_{LP,10}(z)$ and $H_{LP,1}(z)$ plotted using MATLAB are shown below:

$H_{LP,10}(z); \omega_c = 0.3\pi$

Figure title: $H_{LP,1}(z)$; $\omega_c = 0.3\pi$

From these plots we make the following observation – The series connection of 10 1st order IIR lowpass sections achieves a sharper transition band than the single first order filter designed for the same 3 dB cutoff frequency.

Q4.16   Substituting $\omega_o = 0.61\pi$ in Eq. (4.19) we get $\beta = \cos(0.61\pi) = -0.3387$

Substituting $\Delta\omega_{3dB} = 0.15\pi$ in Eq. (4.20) we get $(1+\alpha^2)\cos(0.15\pi) - 2\alpha = 0,$ whose solution yields $\alpha = 1.6319$ and $\alpha = 0.6128$.   .

Substituting the value of β and the first value of α in Eq. (4.18) we arrive at the transfer function of the IIR bandpass transfer function

$$H_{BP,1}(z) = \frac{-0.31595\left(1-z^{-2}\right)}{1+0.89143z^{-1}+1.6319z^{-2}}$$

Substituting the value of β and the second value of α in Eq. (4.18) we arrive at the transfer function of the IIR bandpass transfer function
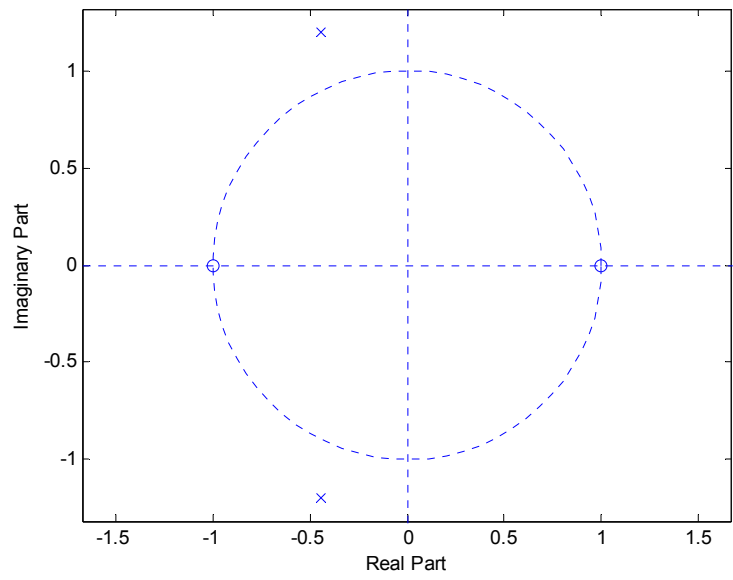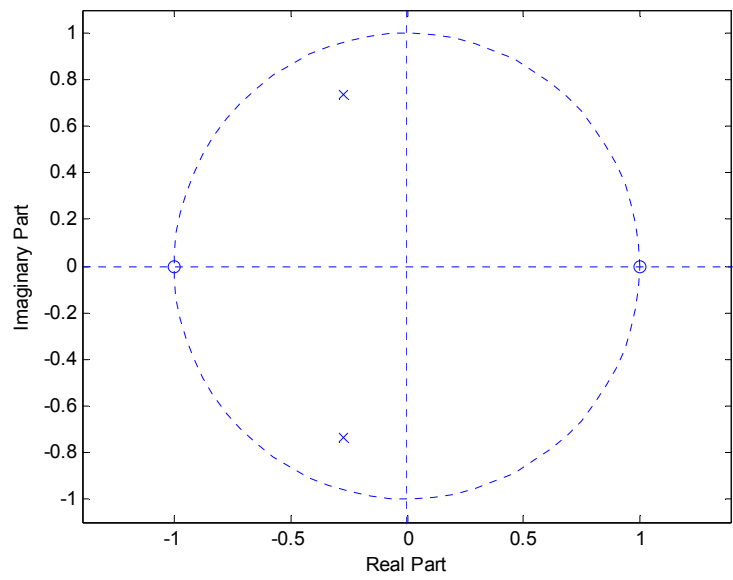
$$H_{BP,2}(z) = \frac{0.19360\left(1-z^{-2}\right)}{1+0.54626z^{-1}+0.6128z^{-2}}$$

Next using the `zplane` command we find the pole locations of $H_{BP,1}(z)$ and $H_{BP,2}(z)$ from which we conclude that the stable transfer function $H_{BP}(z)$ is given by $- H_{BP,2}(z)$.
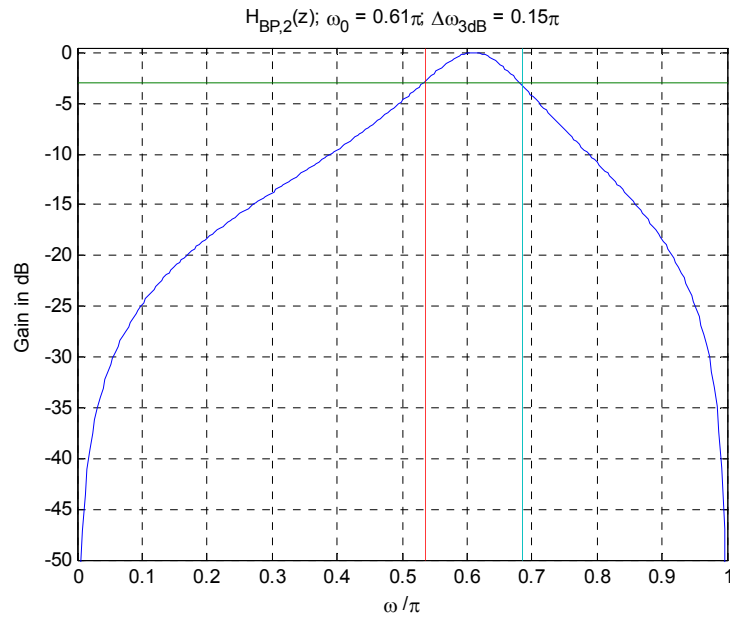
P-Z plot for $H_{BP,1}(z)$

P-Z plot for $H_{BP,2}(z)$

The plot of the gain response of the stable transfer function $H_{BP}(z)$ obtained using MATLAB is shown below:

$H_{BP,2}(z);\ \omega_0 = 0.61\pi,\ \Delta\omega_{3dB} = 0.15\pi$

Using the same value of α and β in Eq. (4.21) we next obtain the transfer function of a stable IIR bandstop filter as

$$H_{BS}(z) \;=\; \frac{0.8064\left(1 + 0.6774z^{-1} + z^{-2}\right)}{1 + 0.54626z^{-1} + 0.6128z^{-2}}$$

The plot of the gain response of the transfer function $H_{BS}(z)$ obtained using MATLAB is shown below:
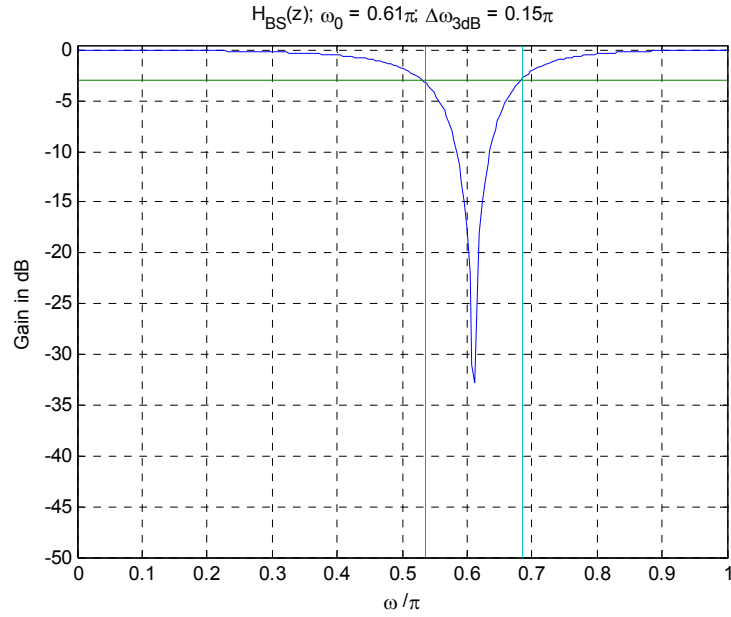
$H_{BS}(z);\ \omega_0 = 0.61\pi;\ \Delta\omega_{3dB} = 0.15\pi$

From these plots we observe that the designed filters **DO** meet the specifications.

A plot of the magnitude response of the sum $H_{BP}(z) + H_{BS}(z)$ obtained using MATLAB is given below:
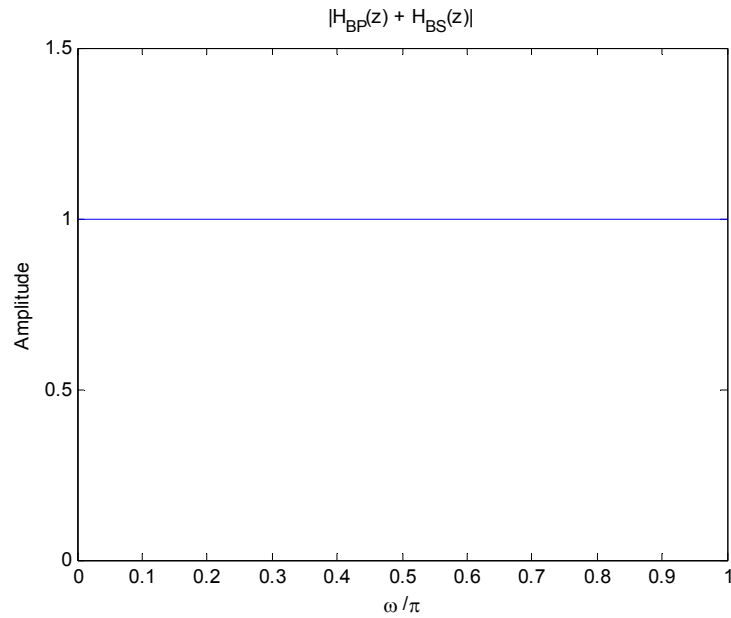
$|H_{BP}(z) + H_{BS}(z)|$

From this plot we observe that the two filters are – Allpass Complementary

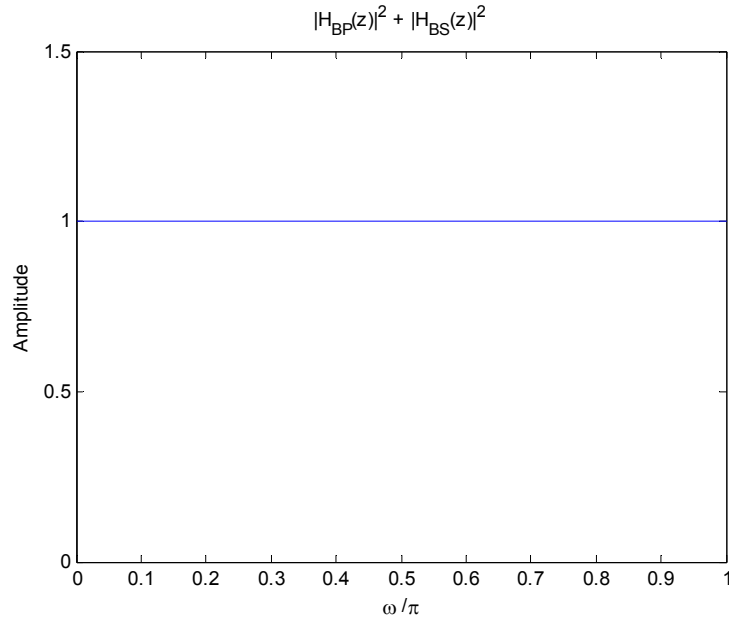A plot of the sum of the square-magnitude responses of $H_{BP}(z)$ and $H_{BS}(z)$ obtained using MATLAB is given below:

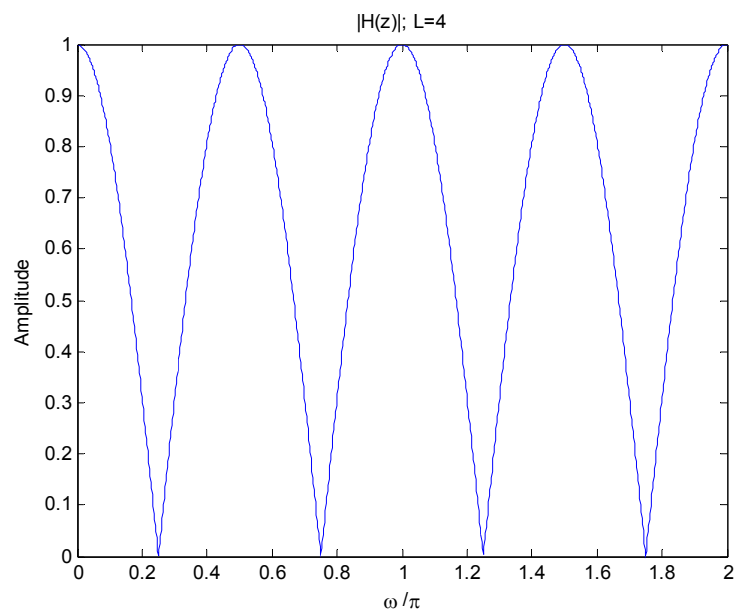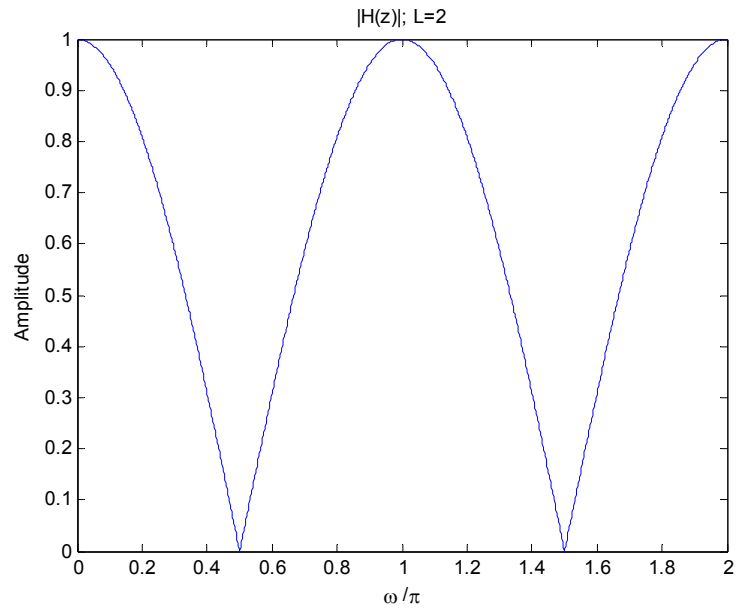$$|H_{BP}(z)|^2 + |H_{BS}(z)|^2$$

From this plot we observe that the two filters are - Power Complementary

Q4.17 The transfer function of a comb filter derived from the prototype FIR lowpass filter of Eq. (4.40) is given by

$$G(z) = H_0(z^L) = 0.5 + 0.5z^{-L}$$

Plots of the magnitude response of the above comb filter for the following values of L are shown below:

|H(z)|; L=7
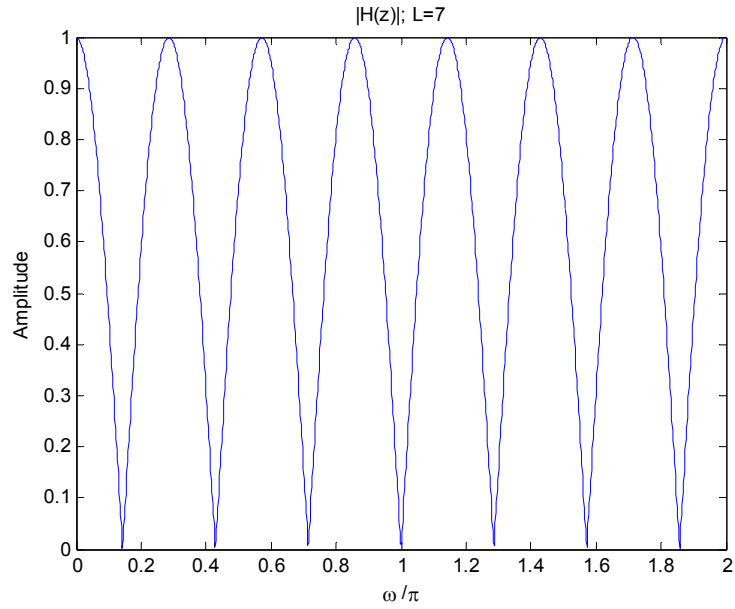
From these plots we observe that the comb filter has __L__ notches at $\omega_k = $ __(2k+1)π/L__ and __L__ peaks at $\omega_k = $ __2kπ/L__, where k = 0, 1, . . ., __L-1__.

Q4.18 The transfer function of a comb filter derived from the prototype FIR highpass filter of Eq. (4.42) with M = 2 is given by

$$G(z) = H_1(z^L) = 0.5 - 0.5z^{-L}$$

Plots of the magnitude response of the above comb filter for the following values of L are shown below –

|H(z)|; L=2

|H(z)|; L=4

|H(z)|; L=7
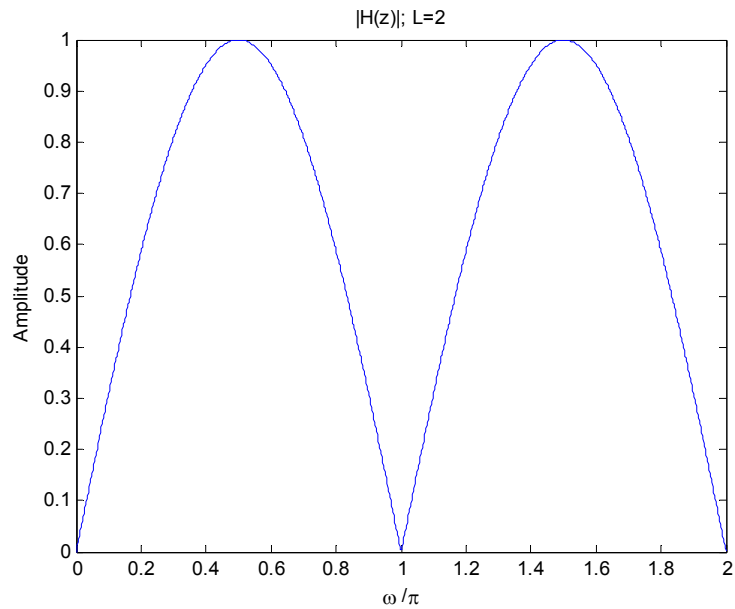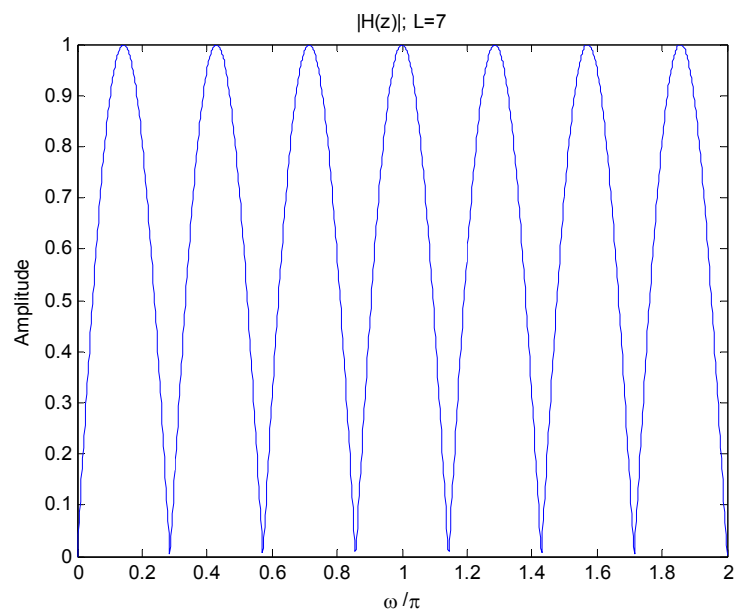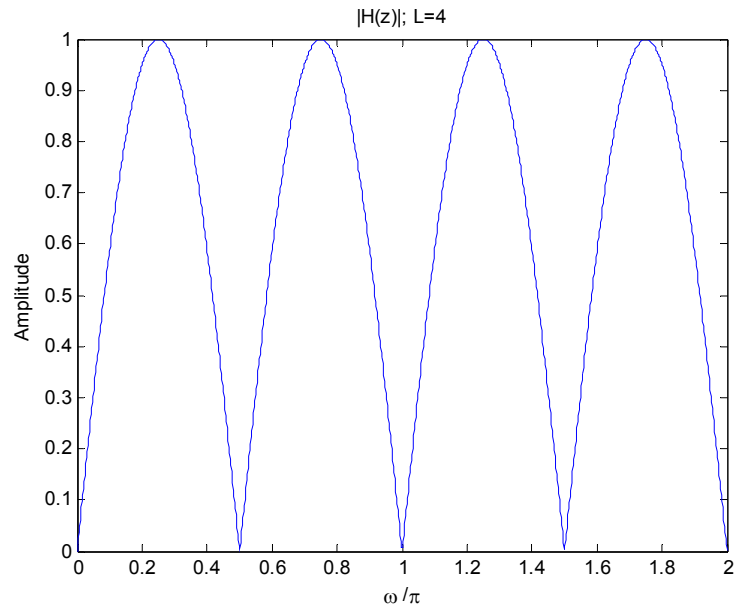
From these plots we observe that the comb filter has $\underline{}L\underline{}$ notches at ω_k = $\underline{}2k\pi/L\underline{}$ and $\underline{}L\underline{}$ peaks at ω_k = $\underline{(2k+1)\pi/L}$.

Q4.19   A copy of Program P4_3 is given below:

```matlab
% Program P4_3
% Zero Locations of Linear Phase FIR Filters
clf;
b = [1 -8.5 30.5 -63];
num1 = [b 81 fliplr(b)];
num2 = [b 81 81 fliplr(b)];
num3 = [b 0 -fliplr(b)];
num4 = [b 81 -81 -fliplr(b)];
n1 = 0:length(num1)-1;
n2 = 0:length(num2)-1;
subplot(2,2,1); stem(n1,num1);
xlabel('Time index n');ylabel('Amplitude'); grid;
title('Type 1 FIR Filter');
subplot(2,2,2); stem(n2,num2);
xlabel('Time index n');ylabel('Amplitude'); grid;
title('Type 2 FIR Filter');
subplot(2,2,3); stem(n1,num3);
xlabel('Time index n');ylabel('Amplitude'); grid;
title('Type 3 FIR Filter');
subplot(2,2,4); stem(n2,num4);
xlabel('Time index n');ylabel('Amplitude'); grid;
title('Type 4 FIR Filter');
pause
subplot(2,2,1); zplane(num1,1);
title('Type 1 FIR Filter');
subplot(2,2,2); zplane(num2,1);
title('Type 2 FIR Filter');
subplot(2,2,3); zplane(num3,1);
title('Type 3 FIR Filter');
subplot(2,2,4); zplane(num4,1);
title('Type 4 FIR Filter');
disp('Zeros of Type 1 FIR Filter are');
disp(roots(num1));
disp('Zeros of Type 2 FIR Filter are');
disp(roots(num2));
disp('Zeros of Type 3 FIR Filter are');
disp(roots(num3));
disp('Zeros of Type 4 FIR Filter are');
disp(roots(num4));
```

The plots of the impulse responses of the four FIR filters generated by running Program P4_3 are given below:

From the plots we make the following observations:

Filter #1 is of length $\underline{9}$ with a <u>symmetric</u> impulse response and is therefore a Type $\underline{1}$ linear-phase FIR filter.

Filter #2 is of length $\underline{10}$ with a <u>symmetric</u> impulse response and is therefore a Type $\underline{2}$ linear-phase FIR filter.

Filter #3 is of length $\underline{9}$ with an <u>antisymmetric</u> impulse response and is therefore a Type $\underline{3}$ linear-phase FIR filter.

Filter #4 is of length $\underline{10}$ with an <u>antisymmetric</u> impulse response and is therefore a Type $\underline{4}$ linear-phase FIR filter.

From the zeros of these filters generated by Program P4_3 we observe that:

Filter #1 has zeros at z =   2.9744,   2.0888, 0.9790 + 1.4110i, 0.9790 - 1.4110i, 0.3319 + 0.4784i, 0.3319 - 0.4784i, 0.4787, 0.3362

Filter #2 has zeros at z = 3.7585 + 1.5147i, 3.7585 - 1.5147i, 0.6733 + 2.6623i, 0.6733 - 2.6623i, -1.0000, 0.0893 + 0.3530i, 0.0893 - 0.3530i, 0.2289 + 0.0922i, 0.2289 - 0.0922i

Filter #3 has zeros at z = 4.7627, 1.6279 + 3.0565i, 1 .6279 - 3.0565i, -1.0000, 1.0000, 0.1357 + 0.2549i, 0.1357 - 0.2549i, 0.2100

Filter #4 has zeros at z = 3.4139, 1.6541 + 1.5813i, 1.6541 - 1.5813i, -0.0733 + 0.9973i, -0.0733 - 0.9973i, 1.0000, 0.3159 + 0.3020i, 0.3159 - 0.3020i, 0.2929
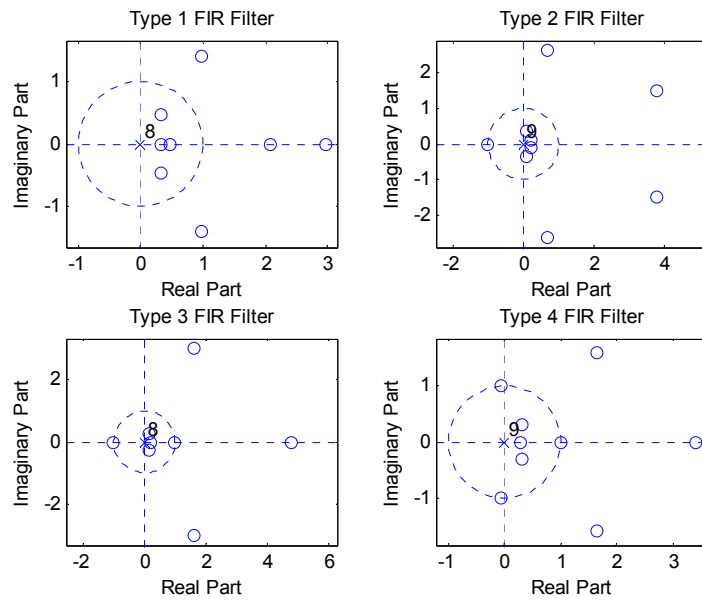
Plots of the phase response of each of these filters obtained using MATLAB are shown below:

Note: These are the UNWRAPPED phases
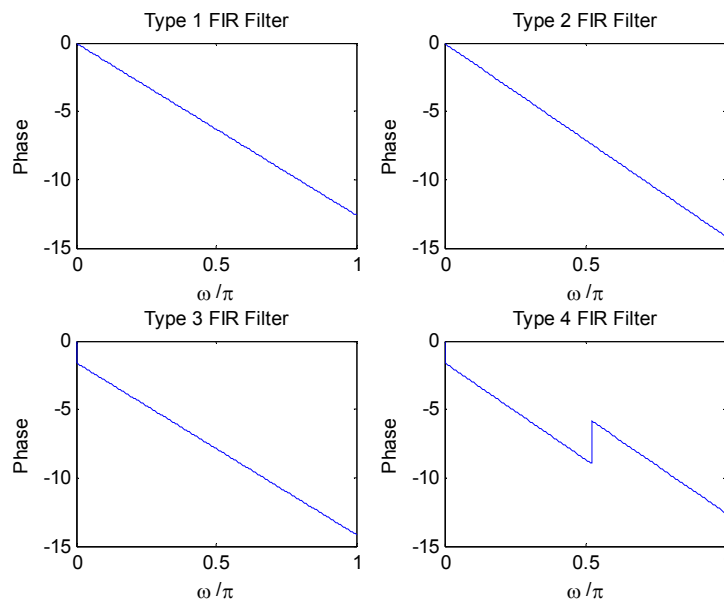
From these plots we conclude that each of these filters have <u>LINEAR (or in the case of the Type 4, generalized LINEAR)</u> phase.

The group delay of Filter # 1 is - 4.0000

The group delay of Filter # 2 is - 4.5000

The group delay of Filter # 3 is - 4

The group delay of Filter # 4 is - 4.5000

Q4.20 The plots of the impulse responses of the four FIR filters generated by running Program P4_3 are given below:

From the plots we make the following observations:

Filter #1 is of length 9 with a <u>symmetric</u> impulse response and is therefore a Type 1 linear-phase FIR filter.
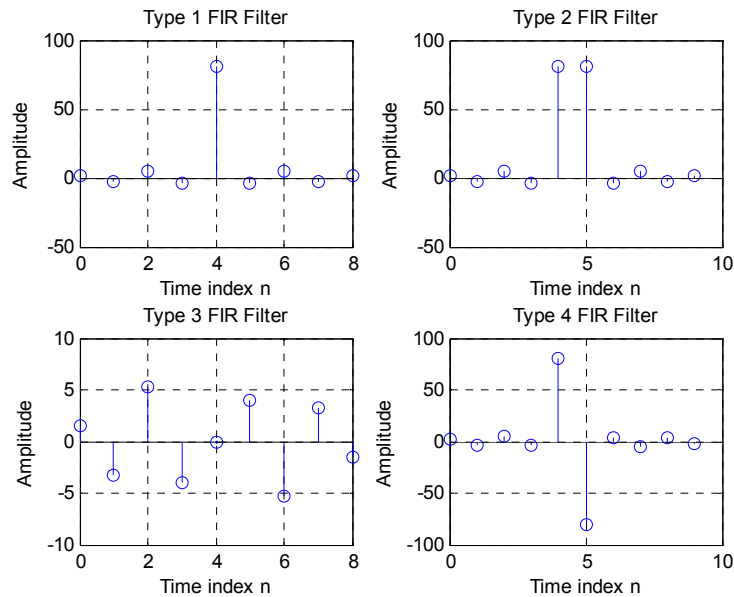
Filter #2 is of length 10 with a <u>symmetric</u> impulse response and is therefore a Type 2 linear-phase FIR filter.

Filter #3 is of length 9 with an <u>antisymmetric</u> impulse response and is therefore a Type 3 linear-phase FIR filter.

Filter #4 is of length 10 with an <u>antisymmetric</u> impulse response and is therefore a Type 4 linear-phase FIR filter.

From the zeros of these filters generated by Program P4_3 we observe that:

Filter #1 has zeros at z =
  $2.3273 + 2.0140i$
  $2.3273 - 2.0140i$
  $-1.2659 + 2.0135i$
  $-1.2659 - 2.0135i$

-0.2238 + 0.3559i
-0.2238 - 0.3559i
 0.2457 + 0.2126i
 0.2457 - 0.2126i

Filter #2 has zeros at z =
  2.5270 + 2.0392i
  2.5270 - 2.0392i
 -1.0101 + 2.1930i
 -1.0101 - 2.1930i
 -1.0000
 -0.1733 + 0.3762i
 -0.1733 - 0.3762i
  0.2397 + 0.1934i
  0.2397 - 0.1934i

Filter #3 has zeros at z =
 -1.0000
  0.2602 + 1.2263i
  0.2602 - 1.2263i
  1.0000
  0.6576 + 0.7534i
  0.6576 - 0.7534i
  0.1655 + 0.7803i
  0.1655 - 0.7803i

Filter #4 has zeros at z =
  2.0841 + 2.0565i
  2.0841 - 2.0565i
 -1.5032 + 1.9960i
 -1.5032 - 1.9960i
  1.0000
 -0.2408 + 0.3197i
 -0.2408 - 0.3197i
  0.2431 + 0.2399i
  0.2431 - 0.2399i

Plots of the phase response of each of these filters obtained using MATLAB are shown below:
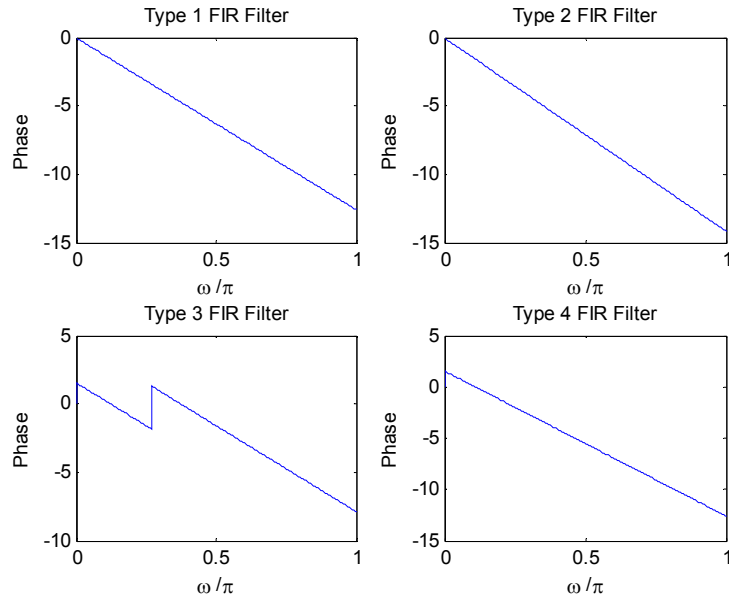
Type 1 FIR Filter   Type 2 FIR Filter
Type 3 FIR Filter   Type 4 FIR Filter

From these plots we conclude that each of these filters have <u>LINEAR, or in the case of the Type 3 filter, Generalized LINEAR</u> phase.

The group delay of Filter # 1 is - 4.0000
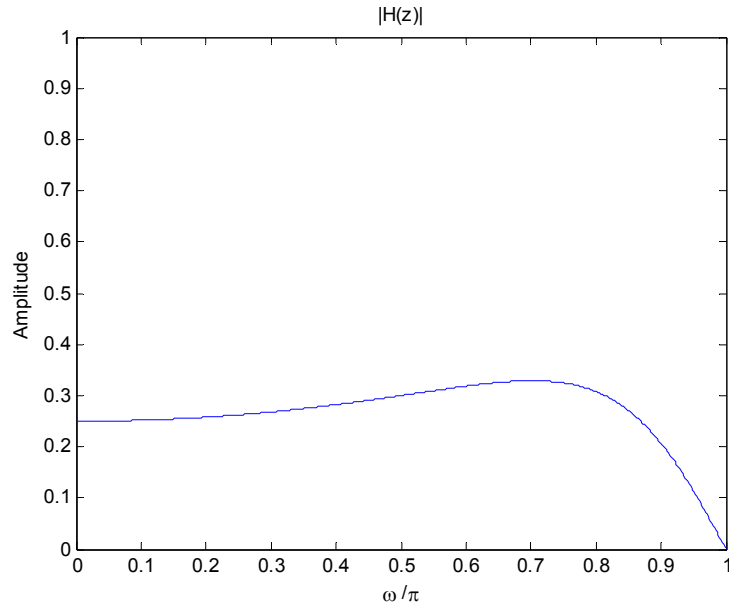
The group delay of Filter # 2 is - 4.5000

The group delay of Filter # 3 is - 4.0000

The group delay of Filter # 4 is - 4.5000

Answers:

Q4.21   A plot of the magnitude response of $H_1(z)$ obtained using MATLAB is shown below:

|H(z)|

From this plot we observe that the magnitude response has a maximum at $\omega = 0.5049\pi$ with a value $= 0.3289$

Using `zplane` we observe that the poles of $H_1(z)$ are <u>OUTSIDE</u> the unit circle at $z = -2$ and $z = -3$. Hence the transfer function IS NOT stable.

Since the maximum value of the magnitude response of $H_1(z)$ is = _____ , we scale $H_1(z)$ by _____ and arrive at a bounded-real transfer function

This question doesn't make any sense. Since $H_1(z)$ has poles outside the unit circle, it cannot be made into a bounded real transfer function by simple scaling (the original author of labex4.doc is confused here). To obtain a bounded real transfer function with the same magnitude as $H_1(z)$, we have to factor $H_1(z) = H_2(z)H_{AP}(z)$ were $H_{AP}(z)$ is allpass and $H_2(z)$ has all poles inside the unit circle. Recall that the transfer function for a first-order allpass section is given by $(-\alpha^* + z^{-1})/(1 - \alpha z^{-1})$. $H_1(z)$ may be factored as follows:

$$
H_1(z) = \frac{1.5(1+z^{-1})}{1+5z^{-1}+6z^{-2}} = \frac{1.5(1+z^{-1})}{(1+3z^{-1})(1+2z^{-1})} = \frac{1.5(1+z^{-1})}{1} \cdot \underbrace{\frac{1}{1+3z^{-1}}}_{\alpha=-3} \cdot \underbrace{\frac{1}{1+2z^{-1}}}_{\alpha=-2}
$$

$$
= \frac{1.5(1+z^{-1})}{1} \cdot \frac{1}{1+3z^{-1}} \cdot \frac{3+z^{-1}}{3+z^{-1}} \cdot \frac{1}{1+2z^{-1}} \cdot \frac{2+z^{-1}}{2+z^{-1}}
$$

$$
= \frac{1.5(1+z^{-1})}{1} \cdot \frac{1}{3+z^{-1}} \cdot \frac{3+z^{-1}}{1+3z^{-1}} \cdot \frac{1}{2+z^{-1}} \cdot \frac{2+z^{-1}}{1+2z^{-1}}
$$

$$
= \underbrace{\frac{1.5(1+z^{-1})}{(3+z^{-1})(2+z^{-1})}}_{H_2(z)} \cdot \underbrace{\frac{(z^{-1}+3)(z^{-1}+2)}{(1+3z^{-1})(1+2z^{-1})}}_{H_{AP}(z)} .
$$

33

Multiplying out on top and bottom, we have

$$H_2(z) = \frac{1.5(1+z^{-1})}{3\left(1+\frac{1}{3}z^{-1}\right)(2)\left(1+\frac{1}{2}z^{-1}\right)} = \frac{\left(\frac{1}{6}\right)\left(\frac{3}{2}\right)\left(1+z^{-1}\right)}{\left(1+\frac{1}{3}z^{-1}\right)\left(1+\frac{1}{2}z^{-1}\right)} = \frac{\frac{1}{4}+\frac{1}{4}z^{-1}}{1+\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2}}$$

and

$$H_{AP}(z) = \frac{\left(3+z^{-1}\right)}{\left(1+3z^{-1}\right)}\cdot\frac{\left(2+z^{-1}\right)}{\left(1+2z^{-1}\right)} = \frac{6+5z^{-1}+z^{-2}}{1+5z^{-1}+6z^{-2}}.$$

Using the Matlab freqz function, you can easily verify that $H_2(z)$ has the same magnitude response as $H_1(z)$, but is a bounded real transfer function. You may also use freqz to easily verify that $H_{AP}(z)$ is allpass.

.

Q4.22   A plot of the magnitude response of $G_1(z)$ obtained using MATLAB is shown below: **NOTE: the errata explain that the factor 1.5 should be deleted from the right hand side of (4.44).**

$|G_1(z)|$
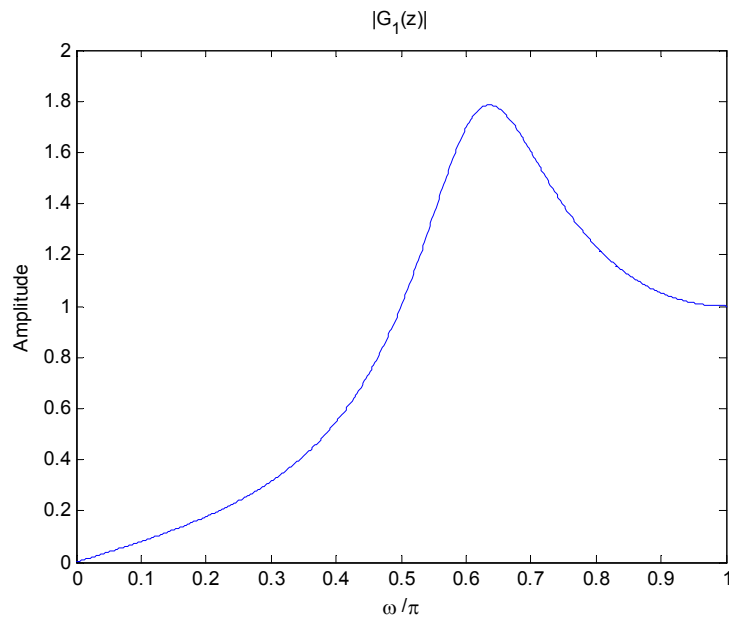
From this plot we observe that the magnitude response has a maximum at $\omega = 0.4599\pi$ with a value $= 1.7854$

Using zplane we observe that the poles of $G_1(z)$ are <u>INSIDE</u> the unit circle and hence the transfer function <u>IS</u> stable.

Since the maximum value of the magnitude response of $G_1(z)$ is $= 1.7854$, we scale $G_1(z)$ by $1/1.7854$ and arrive at a bounded-real transfer function

$$G_2(z) = G_2(z) = \frac{0.5601\left(1 - z^{-1}\right)}{2 + z^{-1} + z^{-2}}$$
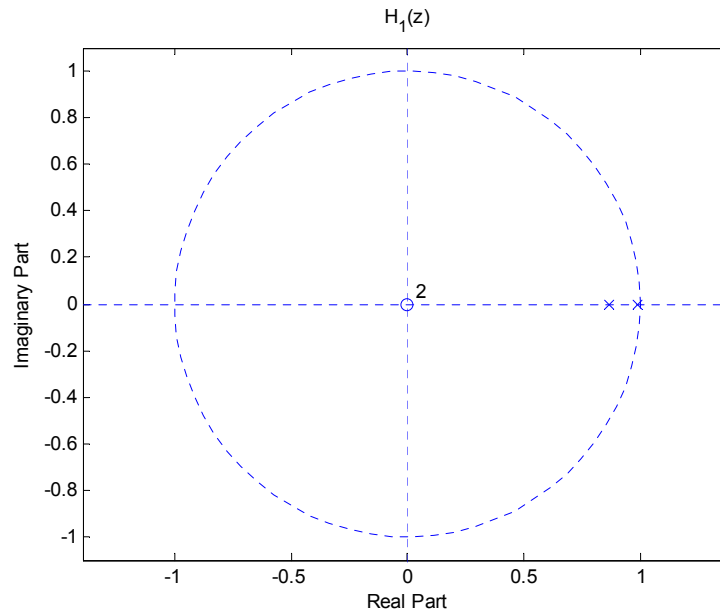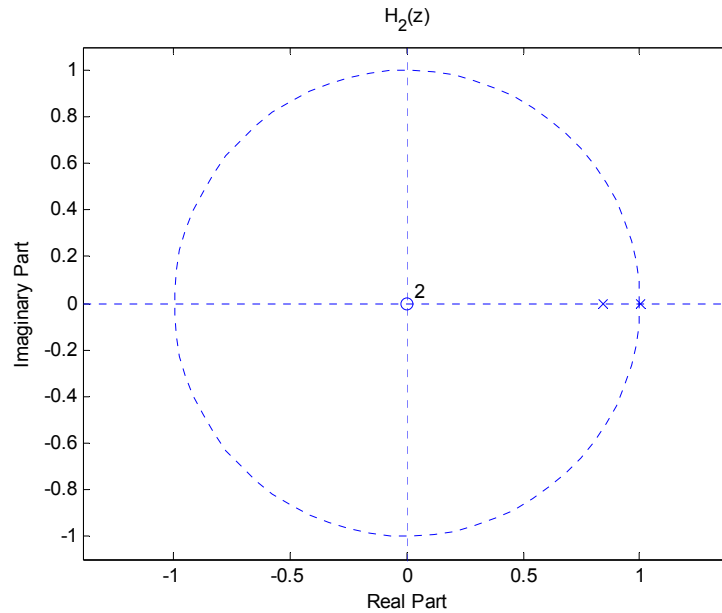
4.3   STABILITY TEST

A copy of Program P4_4 is given below:

```
% Program P4_4
% Stability Test
clf;
den = input('Denominator coefficients = ');
ki = poly2rc(den);
disp('Stability test parameters are');
disp(ki);
```

Answers:

Q4.23   The pole-zero plots of $H_1(z)$ and $H_2(z)$ obtained using zplane are shown below:

$H_2(z)$

From the above pole-zero plots we observe that – It appears that $H_1(z)$ is stable and $H_2(z)$ is unstable. This can be confirmed directly with residuez.

Q4.24 Using Program P4_4 we tested the stability of $H_1(z)$ and arrive at the following stability test parameters $\{k_i\}$:   -0.9989, 0.8500

From these parameters we conclude that $H_1(z)$ is <u>STABLE</u> .

Using Program P4_4 we tested the stability of $H_2(z)$ and arrive at the following stability test parameters $\{k_i\}$: -1.0005, 0.8500

From these parameters we conclude that $H_2(z)$ is <u>UNSTABLE</u> .

Q4.25 Using Program P4_4 we tested the root locations of D(z) and arrive at the following stability test parameters $\{k_i\}$:
   0.9630
   0.8726
   0.6225
   0.2346
   0.0313

From these parameters we conclude that all roots of D(z) are <u>INSIDE</u> the unit circle.

Q4.26   Using Program P4_4 we tested the root locations of D(z) and arrive at the following stability test parameters $\{k_i\}$:

   -0.6087
   0.7958
   0.6742
   0.5938
   0.6000

From these parameters we conclude that all roots of D(z) are <u>INSIDE</u> the unit circle.

Date: 25 October 2007   **Signature**: SOLUTION (Havlicek)