Name: SOLUTION

Section:

Laboratory Exercise 6

DIGITAL FILTER STRUCTURES

6.1 REALIZATION OF FIR TRANSFER FUNCTIONS

Project 6.1 Cascade Realization

Note: program P6_1.m cannot be called directly as suggested in Q6.1 below. This is because `tf2zp` requires the length of the numerator and denominator polynomials to be the same. Thus, it is necessary to MODIFY P6_1.m as shown below.

A copy of the MODIFIED Program P6_1 is given below:

```
% Program P6_1A
% Conversion of a rational transfer function
% to its factored form.
% MODIFIED to make the numerator and denominator coefficient vectors
%   the same length for calling tf2zp.
num = input('Numerator coefficient vector = ');
den = input('Denominator coefficient vector = ');
[b,a] = eqtflength(num,den); % make lengths equal
[z,p,k] = tf2zp(b,a);
sos = zp2sos(z,p,k)
```

Answers:

Q6.1 By running Program P6_1 with `num` = [2 10 23 34 31 16 4] and `den` = [1] we arrive at the following second-order factors:

$$h[0] = 2$$

$$\beta_{11} = 3 \qquad \beta_{21} = 2$$

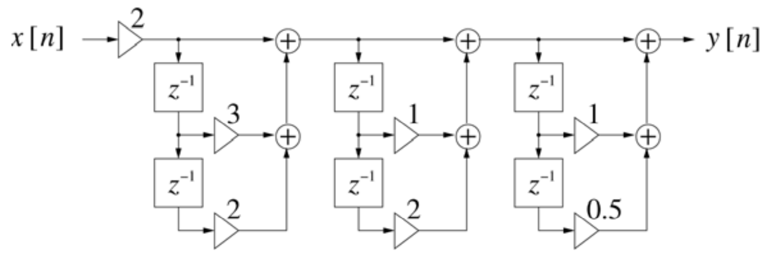
$$\beta_{12} = 1 \qquad \beta_{22} = 2$$

$$\beta_{13} = 1 \qquad \beta_{23} = 0.5$$

In other words, with regards to Eq. (6.3) on p. 92 of the Lab Manual, we have

$$H_1(z) = 2\left(1+3z^{-1}+2z^{-2}\right)\left(1+z^{-1}+2z^{-2}\right)\left(1+z^{-1}+0.5z^{-2}\right)$$

The block-diagram of the cascade realization obtained from these factors is given below:



$H_1(z)$ is NOT a linear-phase transfer function, because the coefficients do not have the required symmetry.

Q6.2 By running Program P6_1 with `num` = [6 31 74 102 74 31 6] and `den` = [1] we arrive at the following second-order factors:

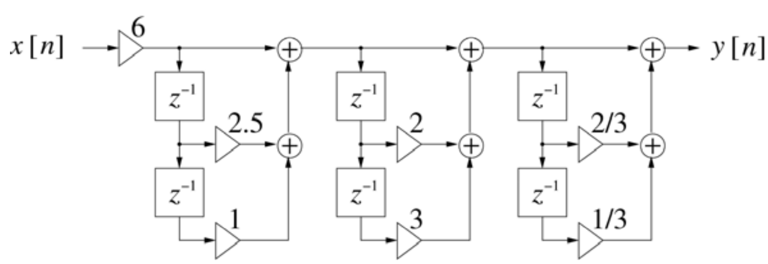
$$h[0] = 6$$

$$\beta_{11} = \frac{15}{6} \qquad \beta_{21} = 1$$

$$\beta_{12} = 2 \qquad \beta_{22} = 3$$

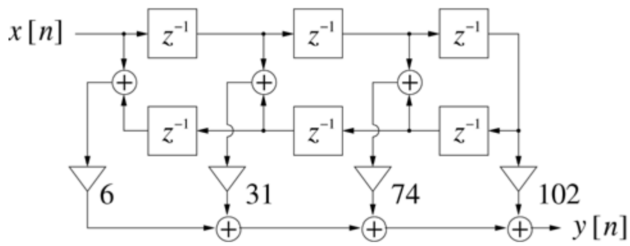
$$\beta_{13} = \frac{2}{3} \qquad \beta_{23} = \frac{1}{3}$$

The block-diagram of the cascade realization obtained from these factors is given below:



$H_2(z)$ is a Type I linear-phase transfer function with odd length and even symmetry.

The block-diagram of the cascade realization of $H_2(z)$ with only 4 multipliers is shown below:



6.2 REALIZATION OF IIR TRANSFER FUNCTIONS

Project 6.2 Cascade Realization

Answers:

Q6.3 By running Program P6_1 with `num` = [3 8 12 7 2 –2] and `den` = [16 24 24 14 5 1] we arrive at the following second-order factors:

The result of running the modified program P6_1 is the following:

```
sos =

    0.1875   -0.0625         0    1.0000    0.5000         0

    1.0000    2.0000    2.0000    1.0000    0.5000    0.2500

    1.0000    1.0000    1.0000    1.0000    0.5000    0.5000
```

In terms of the parameters p_0, α_{jk}, and β_{jk} given in Eq. (6.8) of the Lab Manual, this corresponds to the following:

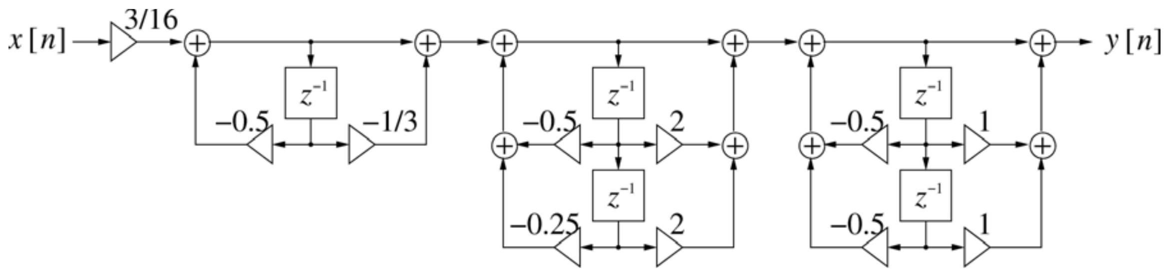
$$p_0 = \frac{3}{16}$$

$$\beta_{11} = -\frac{1}{3} \quad \beta_{21} = 0 \quad \alpha_{11} = \frac{1}{2} \quad \alpha_{21} = 0$$

$$\beta_{12} = 2 \quad \beta_{22} = 2 \quad \alpha_{12} = \frac{1}{2} \quad \alpha_{22} = \frac{1}{4}$$

$$\beta_{13} = 1 \quad \beta_{23} = 1 \quad \alpha_{13} = \frac{1}{2} \quad \alpha_{23} = \frac{1}{2}$$

The block-diagram of the cascade realization obtained from these factors is given below:



Q6.4 By running Program P6_1 with `num` = [2 10 23 34 31 16 4] and `den` = [36 78 87 59 26 7 1] we arrive at the following second-order factors:

The result of running the modified program P6_1 is the following:

```
sos =

    0.0556    0.1667    0.1111    1.0000    0.5000    0.2500

    1.0000    1.0000    2.0000    1.0000    0.6667    0.3333

    1.0000    1.0000    0.5000    1.0000    1.0000    0.3333
```

In terms of the parameters p_0, α_{jk}, and β_{jk} given in Eq. (6.8) of the Lab Manual, this corresponds to the following:

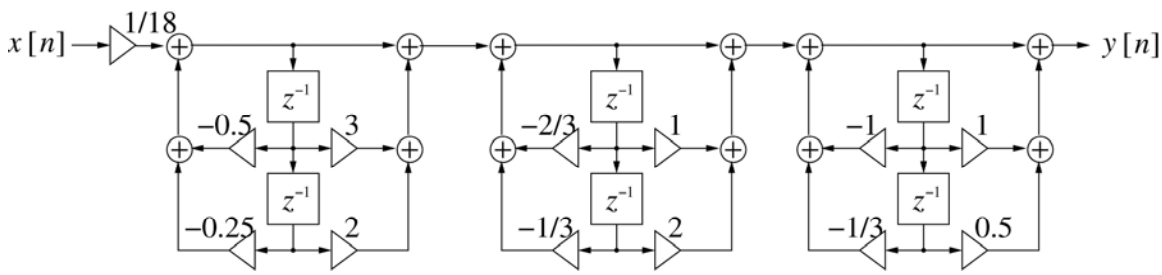
$$p_0 = \frac{1}{18}$$

$$\beta_{11} = 3 \qquad \beta_{21} = 2 \qquad \alpha_{11} = \frac{1}{2} \qquad \alpha_{21} = \frac{1}{4}$$

$$\beta_{12} = 1 \qquad \beta_{22} = 2 \qquad \alpha_{12} = \frac{2}{3} \qquad \alpha_{22} = \frac{1}{3}$$

$$\beta_{13} = 1 \qquad \beta_{23} = \frac{1}{2} \qquad \alpha_{13} = 1 \qquad \alpha_{23} = \frac{1}{3}$$

The block-diagram of the cascade realization obtained from these factors is given below:



A copy of Program P6_2 is given below:

```
% Program P6_2
% Parallel Form Realizations of an IIR Transfer
num = input('Numerator coefficient vector = ');
den = input('Denominator coefficient vector = ');
[r1,p1,k1] = residuez(num,den);
[r2,p2,k2] = residue(num,den);
disp('Parallel Form I')
disp('Residues are');disp(r1);
disp('Poles are at');disp(p1);
disp('Constant value');disp(k1);
disp('Parallel Form II')
disp('Residues are');disp(r2);
disp('Poles are at');disp(p2);
disp('Constant value');disp(k2);
```

Project 6.3 Parallel Realization

Answers:

Q6.5 By running Program P6_2 with `num` = [3 8 12 7 2 –2] and `den` = [16 24 24 14 5 1] we arrive at the partial-fraction expansion of $H_1(z)$ in z^{-1} given by:

For Parallel Form I, the program returns:

```
Parallel Form I
Residues are
  -0.4219 + 0.6201i
  -0.4219 - 0.6201i
   0.3438 - 2.5079i
   0.3438 + 2.5079i
   2.3438
Poles are at
  -0.2500 + 0.6614i
  -0.2500 - 0.6614i
  -0.2500 + 0.4330i
  -0.2500 - 0.4330i
  -0.5000
Constant value
    -2
```

Note that the complex poles occur in conjugate pairs with resides that are also conjugates. Thus, for a pair of conjugate poles at $c+jd$ and $c-jd$ with residues $a+jb$ and $a-jb$, we get a pair of terms in the Partial Fraction Expansion given by (read the help for `residuez` if this isn't clear to you)

$$\frac{a+jb}{1-(c+jd)z^{-1}}+\frac{a-jb}{1-(c-jd)z^{-1}}=\frac{2a-2(ac+bd)z^{-1}}{1-2cz^{-1}+(c^2+d^2)z^{-2}}.$$

For example, for the first pole pair returned for Parallel Form I above, we have $a = -0.4219$, $b = 0.6201$, $c = -0.2500$, and $d = 0.6614$. Thus, the partial fraction expansion in z^{-1} is given by (to within roundoff)

$$
\begin{aligned}
H_1(z) = -2 &+ \frac{2(-0.4219) - 2\left[(-0.4219)(-0.25) + (0.6201)(0.6614)\right]z^{-1}}{1 - 2(-0.25)z^{-1} + \left[\left(-0.25\right)^2 + \left(0.6614\right)^2\right]z^{-2}} \\
&+ \frac{2(0.3438) - 2\left[(0.3438)(-0.25) + (-2.5079)(0.4330)\right]z^{-1}}{1 - 2(-0.25)z^{-1} + \left[\left(-0.25\right)^2 + \left(0.4330\right)^2\right]z^{-2}} \\
&+ \frac{2.3438}{1 + 0.5z^{-1}} \\
= -2 &+ \frac{2.3438}{1 + 0.5z^{-1}} + \frac{-0.8438 - 1.0312z^{-1}}{1 + 0.5z^{-1} + 0.5z^{-2}} + \frac{0.6876 + 2.3437z^{-1}}{1 + 0.5z^{-1} + 0.25z^{-2}}.
\end{aligned}
$$

Comparing this partial fraction expansion to Eq. (6.10) on p. 96 of the Lab Manual, we have the following values for the Parallel Form I parameters:

$\gamma_0 = -2$

$\gamma_{01} = 2.3438$	$\gamma_{11} = 0$	$\alpha_{11} = 0.5$	$\alpha_{21} = 0$
$\gamma_{02} = -0.8438$	$\gamma_{12} = -1.0312$	$\alpha_{12} = 0.5$	$\alpha_{22} = 0.5$
$\gamma_{03} = 0.6876$	$\gamma_{13} = 2.3437$	$\alpha_{13} = 0.5$	$\alpha_{23} = 0.25$

and the partial-fraction expansion of $H_1(z)$ in z given by:

For Parallel Form II, the program returns:

```
Parallel Form II
Residues are
  -0.3047 - 0.4341i
  -0.3047 + 0.4341i
   1.0000 + 0.7758i
   1.0000 - 0.7758i
  -1.1719
Poles are at
  -0.2500 + 0.6614i
  -0.2500 - 0.6614i
  -0.2500 + 0.4330i
  -0.2500 - 0.4330i
  -0.5000
Constant value
    0.1875
```

The complex poles again occur in conjugate pairs with residues that are also conjugates. Read the help for `residue` if you are unclear on how to put this together into the partial fraction expansion. Thus, for a pair of conjugate poles at $c+jd$ and $c-jd$ with residues $a+jb$ and $a-jb$, we get a pair of terms in the partial fraction expansion of the form

$$\frac{a+jb}{z-(c+jd)}+\frac{a-jb}{z-(c-jd)}=\frac{2az-2(ac+bd)}{z^2-2cz+(c^2+d^2)}\cdot\frac{z^{-2}}{z^{-2}}=\frac{2az^{-1}-2(ac+bd)z^{-2}}{1-2cz^{-1}+(c^2+d^2)z^{-2}}.$$

For example, for the first pole pair returned for Parallel Form II above, we have $a=-0.3047$, $b=-0.4341$, $c=-0.2500$, and $d=0.6614$. Thus, the partial fraction expansion in z is given by (to within roundoff)

$$\begin{aligned}H_1(z)=0.1875&+\frac{2(-0.3047)z-2\left[(-0.3047)(-0.25)+(-0.4341)(0.6614)\right]}{z^2-2(-0.25)z+\left[(-0.25)^2+(0.6614)^2\right]}\\&+\frac{2(1)z-2\left[(1)(-0.25)+(0.7758)(0.4330)\right]}{z^2-2(-0.25)z+\left[(-0.25)^2+(0.4330)^2\right]}\\&-\frac{1.1719}{z+0.5}\\=0.1875&-\frac{1.1719}{z+0.5}+\frac{-0.6094z+0.4219}{z^2+0.5z+0.5}+\frac{2z-0.1718}{z^2+0.5z+0.25}.\end{aligned}$$

Multiplying each fraction in this expression times appropriate powers of z^{-1} on top and bottom then gives:

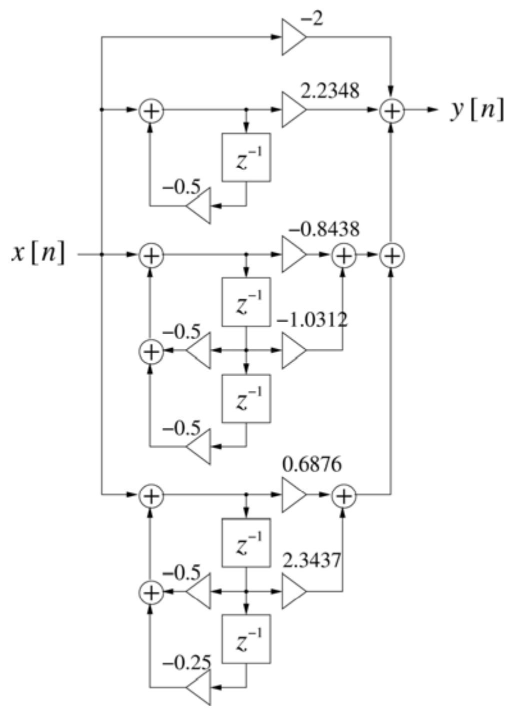
$$\begin{aligned}H_1(z)&=0.1875-\frac{1.1719}{z+0.5}\cdot\frac{z^{-1}}{z^{-1}}+\frac{-0.6094z+0.4219}{z^2+0.5z+0.5}\cdot\frac{z^{-2}}{z^{-2}}+\frac{2z-0.1718}{z^2+0.5z+0.25}\cdot\frac{z^{-2}}{z^{-2}}\\&=0.1875-\frac{1.1719z^{-1}}{1+0.5z^{-1}}+\frac{-0.6094z^{-1}+0.4219z^{-2}}{1+0.5z^{-1}+0.5z^{-2}}+\frac{2z^{-1}-0.1718z^{-2}}{1+0.5z^{-1}+0.25z^{-2}}.\end{aligned}$$

Comparing this partial fraction expansion to Eq. (6.11) on p. 96 of the Lab Manual, we have the following values for the Parallel Form II parameters:

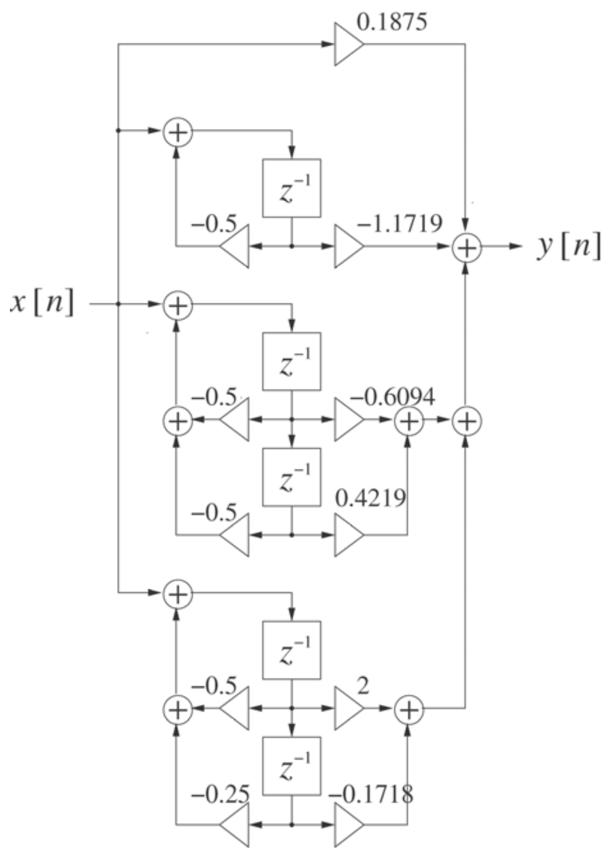
$\delta_0=0.1875$

$\delta_{11}=-1.1719$	$\delta_{21}=0$	$\alpha_{11}=0.5$	$\alpha_{21}=0$
$\delta_{12}=-0.6094$	$\delta_{22}=0.4219$	$\alpha_{12}=0.5$	$\alpha_{22}=0.5$
$\delta_{13}=2$	$\delta_{23}=-0.1718$	$\alpha_{13}=0.5$	$\alpha_{23}=0.25$

The block-diagram of the parallel-form I realization of $H_1(z)$ is thus as indicated below:



The block-diagram of the parallel-form II realization of $H_1(z)$ is thus as indicated below:



Q6.6 By running Program P6_2 with `num` = [2 10 23 34 31 16 4] and `den` = [36 78 87 59 26 7 1] we arrive at the partial-fraction expansion of $H_2(z)$ in z^{-1} given by:

Following the same procedure as in Q6.5, the residues and poles returned for Parallel From I are:

```
Parallel Form I

Residues are

  -0.5952 - 0.7561i
  -0.5952 + 0.7561i
  -0.5556 - 2.2785i
  -0.5556 + 2.2785i
  -0.8214 + 4.3920i
  -0.8214 - 4.3920i

Poles are at

  -0.5000 + 0.2887i
  -0.5000 - 0.2887i
  -0.3333 + 0.4714i
  -0.3333 - 0.4714i
  -0.2500 + 0.4330i
  -0.2500 - 0.4330i
Constant value
     4
```

Plugging into the complex pole pair formulas derived in Q6.5, we have

$$H_2(z) = 4 + \frac{2(-0.5952) - 2\left[(-0.5952)(-0.5) + (-0.7561)(0.2887)\right]z^{-1}}{1 - 2(-0.5)z^{-1} + \left[\left(-0.5\right)^2 + \left(0.2887\right)^2\right]z^{-2}}$$

$$+ \frac{2(-0.5556) - 2\left[(-0.5556)(-0.3333) + (-2.2785)(0.4714)\right]z^{-1}}{1 - 2(-0.3333)z^{-1} + \left[\left(-0.3333\right)^2 + \left(0.4714\right)^2\right]z^{-2}}$$

$$+ \frac{2(-0.8214) - 2\left[(-0.8214)(-0.25) + (4.3920)(0.4330)\right]z^{-1}}{1 - 2(-0.25)z^{-1} + \left[\left(-0.25\right)^2 + \left(0.4330\right)^2\right]z^{-2}}$$

$$= 4 + \frac{-1.1905 - 0.1587z^{-1}}{1 + z^{-1} + 0.3333z^{-2}} + \frac{-1.1111 + 1.7778z^{-1}}{1 + 0.6667z^{-1} + 0.3333z^{-2}} + \frac{-1.6429 - 4.2143z^{-1}}{1 + 0.5z^{-1} + 0.25z^{-2}}.$$

With relation to (6.10) on p. 96 of the Lab Manual, the Parallel Form I parameters are:

$\gamma_0 = 4$

$\gamma_{01} = -1.1905$ $\quad \gamma_{11} = -0.1587$ $\quad \alpha_{11} = 1$ $\quad \alpha_{21} = 0.3333$

$\gamma_{02} = -1.1111$ $\quad \gamma_{12} = -1.7778$ $\quad \alpha_{12} = 0.6667$ $\quad \alpha_{22} = 0.3333$

$\gamma_{03} = -1.6429$ $\quad \gamma_{13} = -4.2143$ $\quad \alpha_{13} = 0.5$ $\quad \alpha_{23} = 0.25$

and the partial-fraction expansion of $H_2(z)$ in z given by:

The residues and poles returned for Parallel From II are:

```
Parallel Form II

Residues are
   0.5159 + 0.2062i
   0.5159 - 0.2062i
   1.2593 + 0.4976i
   1.2593 - 0.4976i
  -1.6964 - 1.4537i
  -1.6964 + 1.4537i

Poles are at
  -0.5000 + 0.2887i
  -0.5000 - 0.2887i
  -0.3333 + 0.4714i
  -0.3333 - 0.4714i
  -0.2500 + 0.4330i
  -0.2500 - 0.4330i

Constant value
    0.0556
```

Plugging into the complex pole pair formulas derived in Q6.5, we have

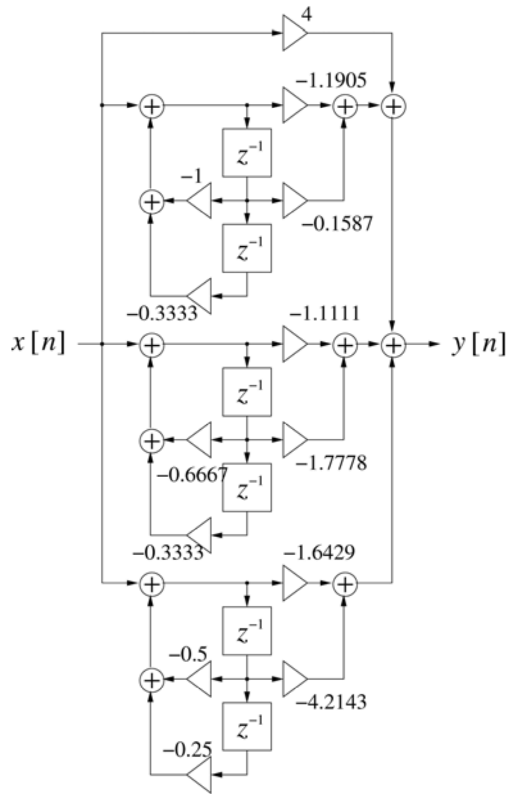
$$H_2(z) = 0.0556 + \frac{2(0.5159)z - 2\left[(0.5159)(-0.5) + (0.2062)(0.2887)\right]}{z^2 - 2(-0.5)z + \left[(-0.5)^2 + (0.2887)^2\right]}$$
$$+ \frac{2(1.2593)z - 2\left[(1.2593)(-0.3333) + (0.4976)(0.4714)\right]}{z^2 - 2(-0.3333)z + \left[(-0.3333)^2 + (0.4714)^2\right]}$$
$$+ \frac{2(-1.6964)z - 2\left[(-1.6964)(-0.25) + (-1.4537)(0.4330)\right]}{z^2 - 2(-0.25)z + \left[(-0.25)^2 + (0.4330)^2\right]}$$
$$= 0.0556 + \frac{1.0317z + 0.3968}{z^2 + z + 0.3333} + \frac{2.5185z + 0.3704}{z^2 + 0.6667z + 0.3333} + \frac{-3.3929z + 0.4107}{z^2 + 0.5z + 0.25}$$
$$= 0.0556 + \frac{1.0317z^{-1} + 0.3968z^{-2}}{1 + z^{-1} + 0.3333z^{-2}} + \frac{2.5185z^{-1} + 0.3704z^{-2}}{1 + 0.6667z^{-1} + 0.3333z^{-2}} + \frac{-3.3929z^{-1} + 0.4107z^{-2}}{1 + 0.5z^{-1} + 0.25z^{-2}}.$$

With relation to (6.11) on p. 96 of the Lab Manual, the Parallel Form II parameters are:

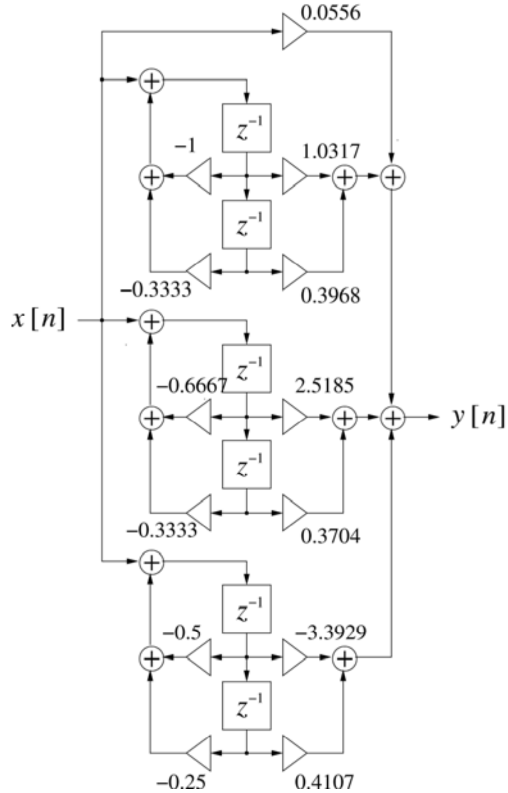
$\delta_0 = 0.0556$

$\delta_{11} = 1.0317$	$\delta_{21} = 0.3968$	$\alpha_{11} = 1$	$\alpha_{21} = 0.3333$
$\delta_{12} = 2.5185$	$\delta_{22} = 0.3704$	$\alpha_{12} = 0.6667$	$\alpha_{22} = 0.3333$
$\delta_{13} = -3.3929$	$\delta_{23} = 0.4107$	$\alpha_{13} = 0.5$	$\alpha_{23} = 0.25$

The block-diagram of the parallel-form I realization of $H_2(z)$ is thus as indicated below:



The block-diagram of the parallel-form II realization of $H_2(z)$ is thus as indicated below:



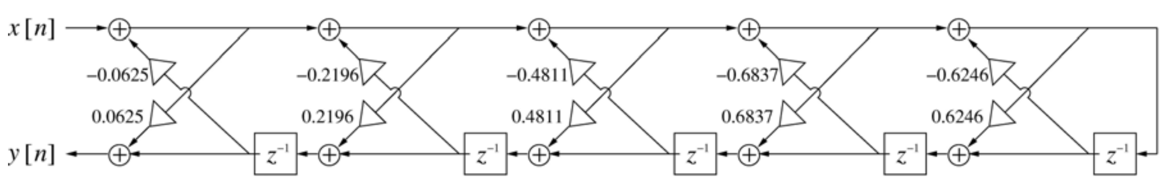
Project 6.4 Realization of an Allpass Transfer function

Answers:

Q6.7 Using Program P4_4 we arrive at the following values of $\{k_i\}$ for $A_5(z)$: All that is required for this problem is to call poly2rc with the coefficients of the denominator polynomial. The first coefficient should be a "1" and it is not here (to make the numbers look nicer). So you may be bothered by the fact that the help for poly2rc says that if d0 isn't one then everything will get scaled. That's true, but it's just fine here because the numerator and denominator both get scaled by 16; in other words, the reflection coefficients are not affected by the scaling. The result of calling ploy2rc is:

```
k(5) = 0.0625    k(4) = 0.2196    k(3) = 0.4811
k(2) = 0.6837    k(1) = 0.6246
```

The block-diagram of the cascaded lattice realization of $A_5(z)$ is thus as shown below:

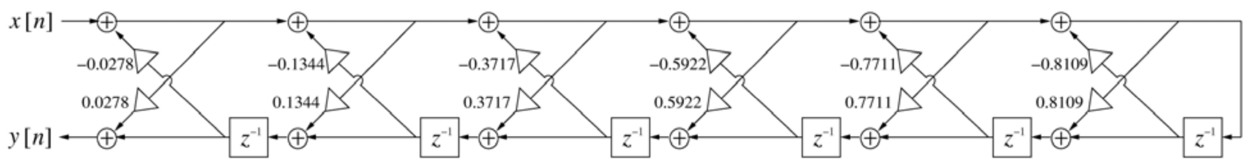


From the values of $\{k_i\}$ we conclude that the transfer function $A_5(z)$ is - STABLE, since $k_i^2 < 1$ for all $1 \le i \le 5$.

Q6.8 Using Program P4_4 we arrive at the following values of $\{k_i\}$ for $A_6(z)$:

```
k(6) = 0.0278    k(5) = 0.1344    k(4) = 0.3717
k(3) = 0.5922    k(2) = 0.7711    k(1) = 0.8109
```

The block-diagram of the cascaded lattice realization of A_6 (z) is thus as shown below.



From the values of $\{k_i\}$ we conclude that the transfer function $A_6(z)$ is – STABLE. All of the reflection coefficients have squared-magnitudes strictly less than unity.

Q6.9 Using `zp2sos` we obtain the following factors of $A_5(z)$:

```
sos =

   0.0625    0.1250         0    1.0000    0.5000         0
   1.0000    2.0000    4.0000    1.0000    0.5000    0.2500
   1.0000    1.0000    2.0000    1.0000    0.5000    0.5000
```

From the above factors we arrive at the decomposition of $A_5(z)$ into its low-order allpass factors as:

$$A_5(z) = \frac{\frac{1}{16}+\frac{1}{8}z^{-1}}{1+\frac{1}{2}z^{-1}} \cdot \frac{1+2z^{-1}+4z^{-2}}{1+\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}} \cdot \frac{1+z^{-1}+2z^{-1}}{1+\frac{1}{2}z^{-1}+\frac{1}{2}z^{-2}}$$

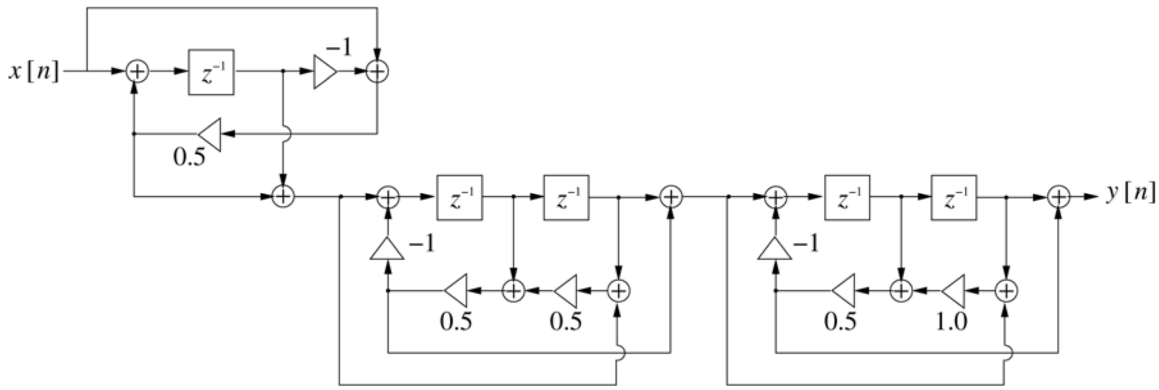
$$= \frac{1}{8} \cdot \frac{\frac{1}{2}+z^{-1}}{1+\frac{1}{2}z^{-1}} \cdot 4 \cdot \frac{\frac{1}{4}+\frac{1}{2}z^{-1}+z^{-2}}{1+\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}} \cdot 2 \cdot \frac{\frac{1}{2}+\frac{1}{2}z^{-1}+z^{-2}}{1+\frac{1}{2}z^{-1}+\frac{1}{2}z^{-2}}$$

$$= \frac{\frac{1}{2}+z^{-1}}{1+\frac{1}{2}z^{-1}} \cdot \frac{\frac{1}{4}+\frac{1}{2}z^{-1}+z^{-2}}{1+\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}} \cdot \frac{\frac{1}{2}+\frac{1}{2}z^{-1}+z^{-2}}{1+\frac{1}{2}z^{-1}+\frac{1}{2}z^{-2}}.$$

The block-diagram of the canonic cascade realization of $A_5(z)$ using Type 1 and 2 allpass sections is thus as indicated below:

NOTE: Since a **canonic** realization is called for, it is required to use the structures Type 1B given in Fig. 6.9(b) on page 97 of the Lab manual and Type 2A given in Fig. 6.10(a) on page 98. The other structures given in the other parts of these figures are **direct**, but not **canonical**, since they use a number of delay elements that exceeds the order of the section.

NOTE 2: In this problem we are **required** to use Type 1 and Type 2 allpass sections. The use of Type 3 sections such as the one shown in Fig. 6.11(a) is therefore not allowed. In this regard, you should contrast (6.14) and (6.15) on p. 97 of the Lab manual. Therefore, for the second section in the expression for A5(z) above, we have d1 = d2 = 0.5. For the third section, we have d1 = 0.5 and d2 = 1.



The total number of multipliers in the final structure is 5.

Q6.10 Using `zp2sos` we obtain the following factors of $A_6(z)$:

```
sos =

   0.0278    0.0556    0.1111    1.0000    0.5000    0.2500

   1.0000    2.0000    3.0000    1.0000    0.6667    0.3333

   1.0000    3.0000    3.0000    1.0000    1.0000    0.3333
```

From the above factors we arrive at the decomposition of $A_6(z)$ into its low-order allpass factors as:

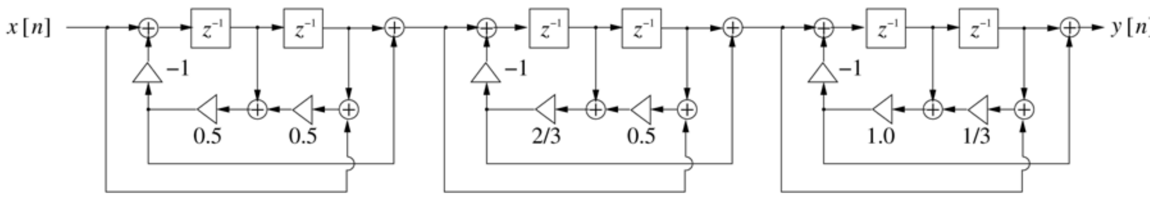
$$A_6(z) = \frac{\frac{1}{36}+\frac{1}{18}z^{-1}+\frac{1}{9}z^{-2}}{1+\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}}\cdot\frac{1+2z^{-1}+3z^{-2}}{1+\frac{2}{3}z^{-1}+\frac{1}{3}z^{-2}}\cdot\frac{1+3z^{-1}+3z^{-1}}{1+z^{-1}+\frac{1}{3}z^{-2}}$$

$$= \frac{1}{9}\cdot\frac{\frac{1}{4}+\frac{1}{2}z^{-1}+z^{-2}}{1+\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}}\cdot 3\cdot\frac{\frac{1}{3}+\frac{2}{3}z^{-1}+z^{-2}}{1+\frac{2}{3}z^{-1}+\frac{1}{3}z^{-2}}\cdot 3\cdot\frac{\frac{1}{3}+z^{-1}+z^{-2}}{1+z^{-1}+\frac{1}{3}z^{-2}}$$

$$= \frac{\frac{1}{4}+\frac{1}{2}z^{-1}+z^{-2}}{1+\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}}\cdot\frac{\frac{1}{3}+\frac{2}{3}z^{-1}+z^{-2}}{1+\frac{2}{3}z^{-1}+\frac{1}{3}z^{-2}}\cdot\frac{\frac{1}{3}+z^{-1}+z^{-2}}{1+z^{-1}+\frac{1}{3}z^{-2}}.$$

The block-diagram of the canonic cascade realization of $A_6(z)$ using Type 2 allpass sections is thus as indicated below:

NOTE again that, because we are required to use Type 2 allpass sections, d1 and d2 cannot be chosen as the explicit coefficients in the numerator and denominator polynomials (contrast (6.14) and (6.15) on p. 97 of the Lab Manual). For the first section, we have d1 = d2 = 0.5. For the second section, a suitable choice is d1 = 2/3 and d2 = 0.5. For the third section, it suffices to take d1 = 1 and d2 = 1/3.



The total number of multipliers in the final structure is 6z.

Project 6.5 Cascaded Lattice Realization of an IIR Transfer function

A copy of Program P6_3 is given below:

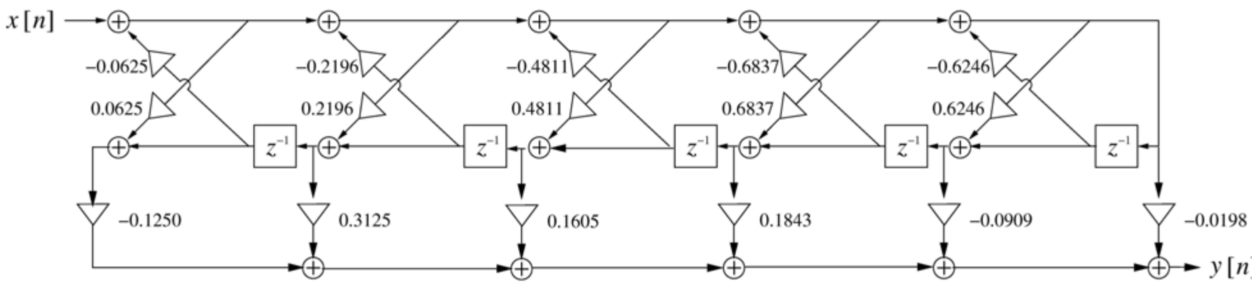
```
% Program P6_3
% Gray-Markel Cascaded Lattice Structure
% k is the lattice parameter vector
% alpha is the vector of feedforward multipliers
format long
% Read in the transfer function coefficients
num = input('Numerator coefficient vector = ');
den = input('Denominator coefficient vector = ');
N = length(den)-1; % Order of denominator polynomial
k = ones(1,N);
a1 = den/den(1);
alpha = num(N+1:-1:1)/den(1);
for ii = N:-1:1,
    alpha(N+2-ii:N+1) = alpha(N+2-ii:N+1)-alpha(N-ii+1)*a1(2:ii+1);
    k(ii) = a1(ii+1);
    a1(1:ii+1) = (a1(1:ii+1)-k(ii)*a1(ii+1:-1:1))/(1-k(ii)*k(ii));
end
disp('Lattice parameters are');disp(k)
disp('Feedforward multipliers are');disp(alpha)
```

Answers:

Q6.11 Using Program P6_3 we arrive at the lattice parameters and the feed-forward multiplier coefficients of the Gray-Markel realization of the causal IIR transfer function $H_1(z)$ of Q6.3 as given below:

```
Lattice parameters are

Columns 1 through 4
 0.62459686089013  0.68373782742919  0.48111942348398  0.21960784313725

Column 5
 0.06250000000000

Feedforward multipliers are

Columns 1 through 4
-0.12500000000000  0.31250000000000  0.16053921568627  0.18430047140849

Columns 5 through 6
  -0.09085169508677 -0.01982100623522
```

From these parameters we obtain the block-diagram of the corresponding Gray-Markel structure as given below:

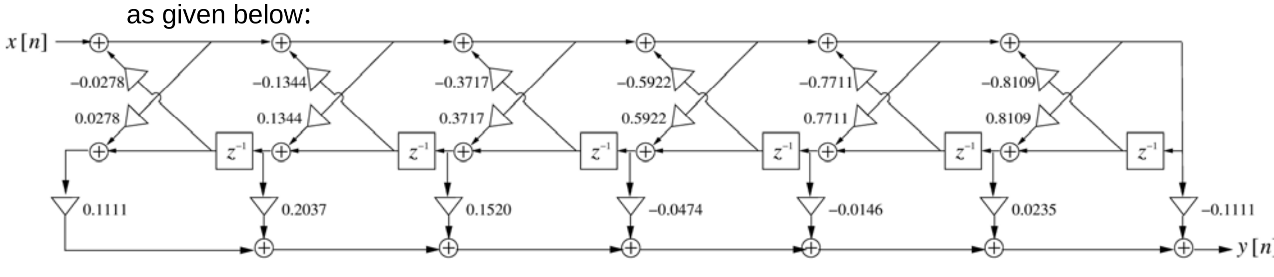


From the lattice parameters obtained using Program P6_3 we conclude that the transfer function $H_1(z)$ is – STABLE, since all the lattice parameters have squared magnitudes strictly less than unity.

Q6.12 Using Program P6_3 we arrive at the lattice parameters and the feed-forward multiplier coefficients of the Gray-Markel realization of the causal IIR transfer function $H_2(z)$ of Q6.4 as given below:

```
Lattice parameters are

  Columns 1 through 4
   0.81093584641352 0.77112772506402 0.59215187769984 0.37169052478550

  Columns 5 through 6
   0.13436293436293 0.02777777777778

Feedforward multipliers are

  Columns 1 through 4
   0.11111111111111 0.20370370370370  0.15199485199485 -0.04739265773254

  Columns 5 through 7
  -0.01456452038379 0.02345313662512 -0.01112037033486
```

From these parameters we obtain the block-diagram of the corresponding Gray-Markel structure as given below:



From the lattice parameters obtained using Program P6_3 we conclude that the transfer function $H_2(z)$ is – STABLE, since all the lattice parameters have squares strictly less than unity in magnitude.

Q6.13 The MATLAB program to develop the Gray-Markel realization of a causal IIR transfer function using the function `tf2latc` is given below:

```
% Program P6_4
% Gray-Markel Cascaded Lattice Structure using tf2latc.
% k is the lattice parameter vector
% alpha is the vector of feedforward multipliers
% Program also computes the inversion of the lattice/ladder vectors.
format long
% Read in the transfer function coefficients
num = input('Numerator coefficient vector = ');
den = input('Denominator coefficient vector = ');
num = num/den(1);  % normalize upstairs and down by d0.
den = den/den(1);
% here is the lattice/ladder realization from the transfer fcn:
[k,alpha] = tf2latc(num,den)
% now check inversion
disp('Check of Lattice/Ladder Inversion:');
[num2,den2] = latc2tf(k,alpha)
```

Using this program we arrive at the lattice parameters and the feed-forward multiplier coefficients (vectors `k` and `alpha`) of the Gray-Markel realization of the transfer function $H_1(z)$ of Q6.3 as given below:

```
k =

   0.62459686089013
   0.68373782742919
   0.48111942348398
   0.21960784313725
   0.06250000000000


alpha =

  -0.01982100623522
  -0.09085169508677
   0.18430047140849
   0.16053921568627
   0.31250000000000
  -0.12500000000000
```

The parameters obtained using this program are THE SAME as those obtained in Q6.11.

Using the function `latc2tf` we obtain the following transfer function from the vectors `k` and `alpha`:

```
num2 =
  Columns 1 through 4
   0.18750000000000   0.50000000000000   0.75000000000000   0.43750000000000
  Columns 5 through 6
   0.12500000000000  -0.12500000000000

den2 =
  Columns 1 through 4
   1.00000000000000   1.50000000000000   1.50000000000000   0.87500000000000
  Columns 5 through 6
   0.31250000000000   0.06250000000000
```

```
>> 16*num2
ans =
  Columns 1 through 4
   3.00000000000000   8.00000000000000  12.00000000000000   7.00000000000000
  Columns 5 through 6
   2.00000000000000  -2.00000000000000

>> 16*den2
ans =
  Columns 1 through 4
  16.00000000000000  24.00000000000000  24.00000000000000  14.00000000000000
  Columns 5 through 6
   5.00000000000000   1.00000000000000
```

The transfer function obtained is EQUIVALENT to $H_1(z)$ of Q6.3; as demonstrated above the numerator and denominator coefficient vectors returned by latc2tf are equal to 1/16 times the values shown in (6.27).

Q6.14 Using this program we arrive at the lattice parameters and the feed-forward multiplier coefficients (vectors `k` and `alpha`) of the Gray-Markel realization of the transfer function $H_2(z)$ of Q6.4 as given below:

```
k =

   0.81093584641352
   0.77112772506402
   0.59215187769984
   0.37169052478550
   0.13436293436293
   0.02777777777778

alpha =

  -0.01112037033486
   0.02345313662512
  -0.01456452038379
  -0.04739265773254
   0.15199485199485
   0.20370370370370
   0.11111111111111
```

The parameters obtained using this program are THE SAME as those obtained in Q6.12.

Using the function `latc2tf` we obtain the following transfer function from the vectors `k` and `alpha`:

```
num2 =
  Columns 1 through 4
   0.05555555555556 0.27777777777778 0.63888888888889 0.94444444444444
  Columns 5 through 7
   0.86111111111111 0.44444444444444 0.11111111111111

den2 =
  Columns 1 through 4
   1.00000000000000 2.16666666666667 2.41666666666667 1.63888888888889
  Columns 5 through 7
```

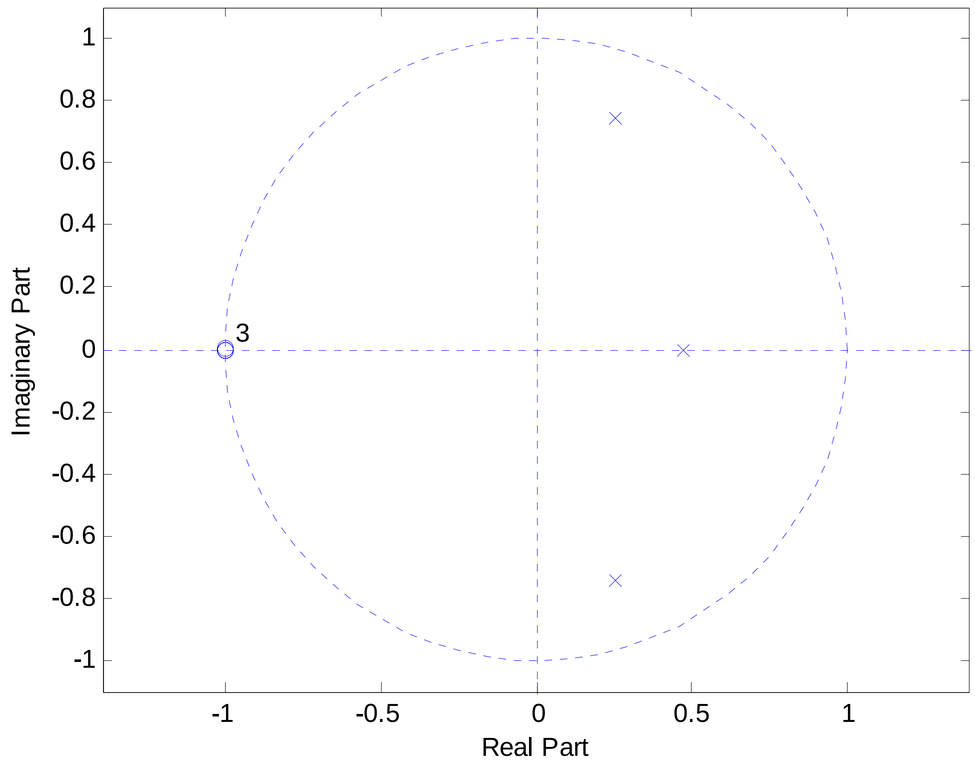
```
   0.72222222222222 0.19444444444444 0.02777777777778

>> 36*num2
ans =
  Columns 1 through 4
   2.00000000000000  10.00000000000000  22.99999999999999
33.99999999999999

  Columns 5 through 7
  31.00000000000000  16.00000000000000   4.00000000000000

>> 36*den2

ans =
  Columns 1 through 4
  36.00000000000000  77.99999999999997  87.00000000000000
58.99999999999999

  Columns 5 through 7
  25.99999999999999   7.00000000000000   1.00000000000000
```

The transfer function obtained is EQUIVALENT to $H_2(z)$ of Q6.4; the numerator and denominator coefficients returned by latc2tf in this question are equal to 1/36 times the original ones appearing in (6.28).

Project 6.6 Parallel Allpass Realization of an IIR Transfer function

Answers:

Q6.15 Using `zplane` we obtain the pole-zero plot of `G(z)` as shown below:



Next using `roots` we obtain the pole locations of `G(z)` as given below:

```
>> p2 = roots(den)

p2 =

  0.25221914966490 + 0.74515855654836i
  0.25221914966490 - 0.74515855654836i
  0.47166170067020

>> Theta = angle(p2)*180/pi

Theta =

  71.30021430520364
 -71.30021430520364
                  0
```

Making use of the pole-alteration property we thus arrive at the two allpass sections $A_0(z)$ and $A_1(z)$ as given below:

Ordering the poles by increasing angle implies that the pair of conjugate poles should be associated with $A_0(z)$, whereas the real pole should be associated with $A_1(z)$. Thus, we have

$$\text{denominator}\{A_0(z)\} = \left[1-\left(0.2522+j0.7452\right)z^{-1}\right]\left[1-\left(0.2522-j0.7452\right)z^{-1}\right]$$
$$= 1-0.5044z^{-1}+0.6189z^{-2},$$

$$\text{denominator}\{A_1(z)\} = 1-0.4717z^{-1}.$$

The numerator polynomials for $A_0(z)$ and $A_1(z)$ then follow from the allpass property; i.e., they must be the mirror image polynomials of the respective denominator polynomials. We have

$$A_0(z) = \frac{0.6189-0.5044z^{-1}+z^{-2}}{1-0.5044z^{-1}+0.6189z^{-2}},$$

$$A_1(z) = \frac{-0.4714+z^{-1}}{1-0.4714z^{-1}}.$$

The power-complementary transfer function $H(z)$ is therefore given by

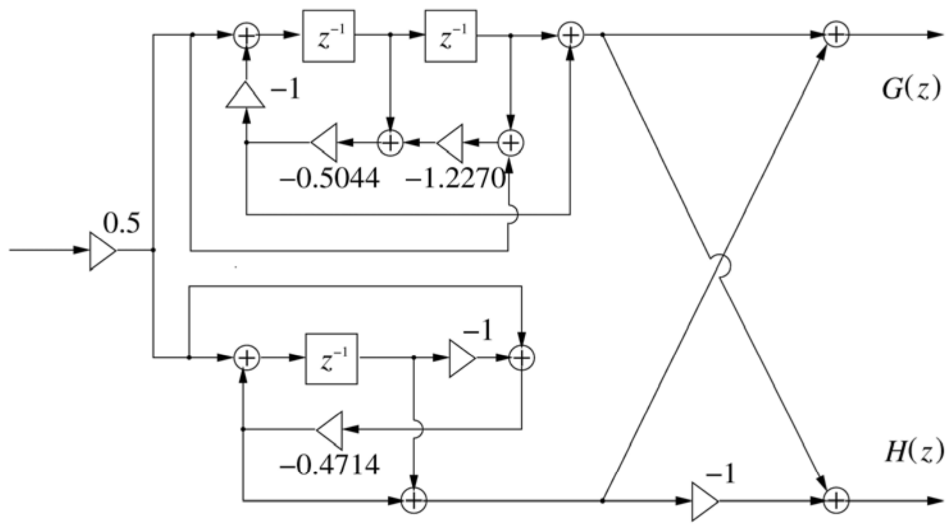
$$H(z) = \frac{1}{2}\left\{A_0(z)-A_1(z)\right\}$$
$$= \frac{1}{2}\left\{\frac{0.6189-0.5044z^{-1}+z^{-2}}{1-0.5044z^{-1}+0.6189z^{-2}} - \frac{-0.4714+z^{-1}}{1-0.4714z^{-1}}\right\}$$
$$= \frac{0.5453-1.01713z^{-1}+1.01713z^{-2}-0.5453z^{-3}}{1-0.9761z^{-1}+0.8568z^{-2}-0.2919z^{-2}}.$$

The order of $A_0(z)$ is – N=2.

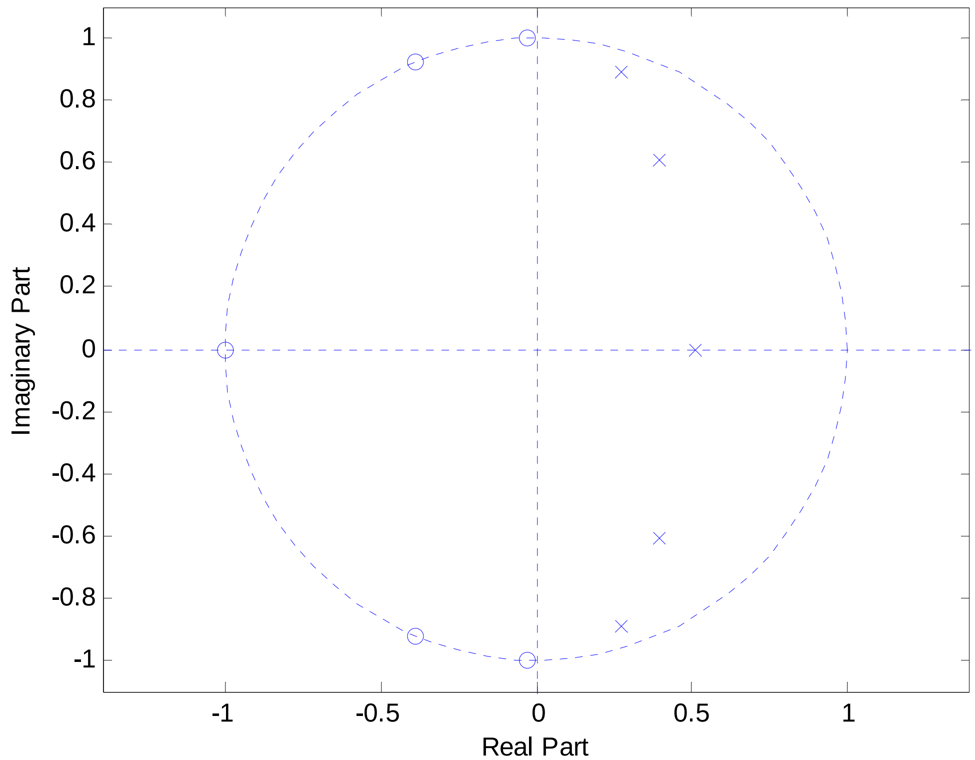
The order of $A_1(z)$ is –N=1.

The block-diagram of a 3-multiplier realization of G(z) and H(z) using Type 1 and Type 2 allpass structures is as indicated below:

From the numerator of A0(z), it follows that d1 = -0.5044. Comparing the above expression for A0(z) with (6.14) on p. 97 of the Lab manual, we then solve for d2 = -1.2270. The value of d1 for A1(z) may be obtained explicitly from the above expression for A1(z).



Q6.16 Using `zplane` we obtain the pole-zero plot of `G(z)` as shown below:



Next using `roots` we obtain the pole locations of `G(z)` as given below:

```
>> p = roots(den)

p =

  0.27615462038702 + 0.89071457007727i
  0.27615462038702 - 0.89071457007727i
  0.39360906693476 + 0.61015746323637i
  0.39360906693476 - 0.61015746323637i
  0.51037262535644
```

Making use of the pole-alteration property we thus arrive at the two allpass sections $A_0(z)$ and $A_1(z)$ as given below:

The angles of the poles are given by

```
>> Theta=angle(p)*130/pi

Theta =

  52.55944857977081
 -52.55944857977081
  41.29245667713326
 -41.29245667713326
                  0
```

This implies that the conjugate pole pair at ~ 53 deg should be combined with the real pole for A0(z), whereas the conjugate pole pair at ~ 41 deg should be combined for A1(z). For the denominators of A0(z) and A1(z) we therefore have

$$\text{denominator}\{A_0(z)\} = \left[1-(0.2762+j0.8907)z^{-1}\right]\left[1-(0.2762-j0.8907)z^{-1}\right]\left(1-0.5104z^{-1}\right),$$
$$\text{denominator}\{A_1(z)\} = \left[1-(0.3936+j0.6102)z^{-1}\right]\left[1-(0.3936-j0.6102)z^{-1}\right].$$

The numerators of A0(z) and A1(z) may then be solved as the mirror image polynomials of the respective denominators. We have

$$A_0(z) = \frac{0.8696-0.5523z^{-1}+z^{-2}}{1-0.5523z^{-1}+0.8696z^{-2}} \cdot \frac{-0.5104+z^{-1}}{1-0.5104z^{-1}},$$
$$A_1(z) = \frac{0.5272-0.7872z^{-1}+z^{-2}}{1-0.7872z^{-1}+0.5272z^{-2}}.$$

The power-complementary transfer function H(z) is therefore given by

$$H(z) = \frac{1}{2}\left\{\frac{0.8696-0.5523z^{-1}+z^{-2}}{1-0.5523z^{-1}+0.8696z^{-2}} \cdot \frac{-0.5104+z^{-1}}{1-0.5104z^{-1}} - \frac{0.5272-0.7872z^{-1}+z^{-2}}{1-0.7872z^{-1}+0.5272z^{-2}}\right\}.$$

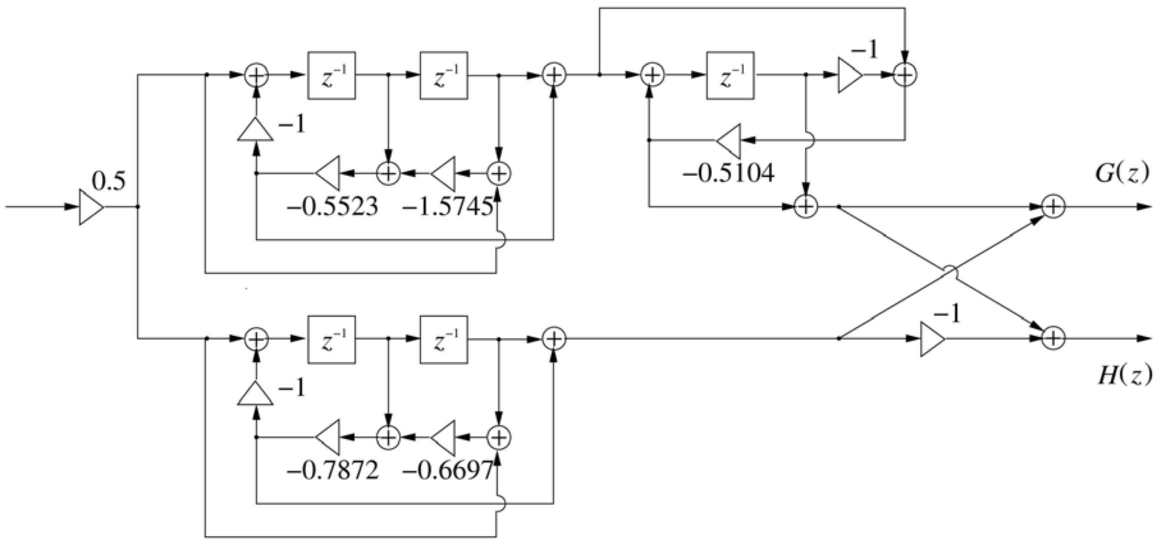
The order of $A_0(z)$ is – N=3.

The order of $A_1(z)$ is – N=2.

The block-diagram of a 5-multiplier realization of G(z) and H(z) using Type 1 and Type 2 allpass structures is as indicated below:

For the first section of A0, we obtain directly from (6.14) on p. 97 of the lab manual that d1 = -0.5523. Solving (6.14) for d2, we obtain d2 = -1.5745. Comparing the above expression for A0(z) to (6.13) on p. 97 of the Lab manual, we obtain for the second section of A0 (the first-order section) that d1 = -0.5104.

Similarly, from (6.14) on p. 97 of the Lab manual and the above expression for A1(z), we have for A1(z) that d1 = -0.7872 and d2 = -0.6697. Thus, the required block diagram is given by



Date: **10 November 2007** **Signature**: **Havlicek**