ECE 5273
Test 1

Tuesday, March 25, 2025
4:30 PM - 5:45 PM

Spring 2025 Name: SOLUTION

Dr. Havlicek Student Num: _____

Directions: This is an open notes test. You may use the official course lecture notes and a calculator. Other materials are not allowed. You have 75 minutes to complete the test. All work must be your own.

SHOW ALL OF YOUR WORK for maximum partial credit!

GOOD LUCK!

SCORE:

1. (20) _____
2. (20) _____
3. (20) _____
4. (20) _____
5. (20) _____

_____

TOTAL (100):

_____

On my honor, I affirm that I have neither given nor received inappropriate aid in the completion of this test.

Name: _____ Date: _____

1. **20 pts.** True or False. Mark *True* only if the statement is **always** true.

TRUE	FALSE	
X		(a) **2 pts.** X-ray computed tomography (CT scan) is an example of absorption imaging. NOTES p. 1.25
	X	(b) **2 pts.** If two digital images have the same histogram, then they are identical up to a point operation. NOTES p. 2.17
	X	(c) **2 pts.** For histogram flattening (equalization), it is important to define the reference point to be the center of the window so that the output image is not shifted. NOTES p. 3.38 There is not a window in Hist EQ
	X	(d) **2 pts.** Frame differencing is a simple but effective technique for smoothing noise in digital video frames. NOTES 3.50, 3.56
	X	(e) **2 pts.** The main purpose of the binary CLOSE filter (which performs dilation first) is to enlarge the objects in an image. NOTES p. 2.88
X		(f) **2 pts.** The discrete-space Fourier transform (DSFT) of the *Peppers* image is periodic. NOTES p. 4.107
X		(g) **2 pts.** For a practical digital image $\mathbf{I}$, the 2D DFT $\widetilde{\mathbf{I}}$ is given by equally spaced samples of the DSFT $\widetilde{\mathbf{I}}_D$. NOTES p. 4.108
X		(h) **2 pts.** If $\mathbf{I}_C$ is a true Gaussian optical image, then the digital image $\mathbf{I}$ obtained with a digital camera is aliased. NOTES p. 133
X		(i) **2 pts.** In the "pinhole" camera model we have used, straight lines in the 3D world always project to straight lines on the 2D focal plane. NOTES p. 1.47
OH MY!		(j) **2 pts.** Greenland and Canada are fighting each other to see who will become the 51'st state.

2. **20 pts.** Consider the 4×4 image $\mathbf{I}$ shown below, where the allowable range of gray levels is $0 \leq I(i,j) \leq 15$:

$$\mathbf{I} = \begin{array}{|c|c|c|c|} \hline \mathbf{4} & \mathbf{5} & \mathbf{9} & \mathbf{7} \\ \hline \mathbf{6} & \mathbf{10} & \mathbf{8} & \mathbf{9} \\ \hline \mathbf{7} & \mathbf{10} & \mathbf{11} & \mathbf{8} \\ \hline \mathbf{4} & \mathbf{6} & \mathbf{5} & \mathbf{11} \\ \hline \end{array}$$

Construct a new image $\mathbf{K}$ by applying the histogram shaping algorithm to make the histogram more "ramp-like." The **desired** histogram shape is given by:

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$\mathrm{H}_{\mathbf{K}}(k)$	1	0	0	2	0	0	0	3	0	0	0	4	0	0	0	6

Show the new image $\mathbf{K}$ and its histogram $\mathrm{H}_{\mathbf{K}}$ in the spaces provided below.

$$\mathbf{K} = \begin{array}{|c|c|c|c|} \hline 3 & 7 & 15 & 11 \\ \hline 7 & 15 & 11 & 15 \\ \hline 11 & 15 & 15 & 11 \\ \hline 3 & 7 & 7 & 15 \\ \hline \end{array}$$

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$\mathrm{H}_{\mathbf{K}}(k)$	0	0	0	2	0	0	0	4	0	0	0	4	0	0	0	6

Work space is provided on the next page.

Workspace for Problem 2:

$J =$

2/16	4/16	12/16	8/16
6/16	14/16	10/16	12/16
9/16	14/16	1	10/16
2/16	6/16	4/16	1

$= P_I\{I(m,n)\}$

For I:

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
H(k)	0	0	0	0	2	2	2	2	2	2	2	2	0	0	0	0
p(k)	0	0	0	0	2/16	2/16	2/16	2/16	2/16	2/16	2/16	2/16	0	0	0	0
P(k)	0	0	0	0	2/16	4/16	6/16	8/16	10/16	12/16	14/16	16/16	1	1	1	1

DESIRED FOR K:

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
H(k)	1	0	0	2	0	0	0	3	0	0	0	4	0	0	0	6
p(k)	1/16	0	0	2/16	0	0	0	3/16	0	0	0	4/16	0	0	0	6/16
P(k)	1/16	1/16	1/16	3/16	3/16	3/16	3/16	6/16	6/16	6/16	6/16	10/16	10/16	10/16	10/16	1

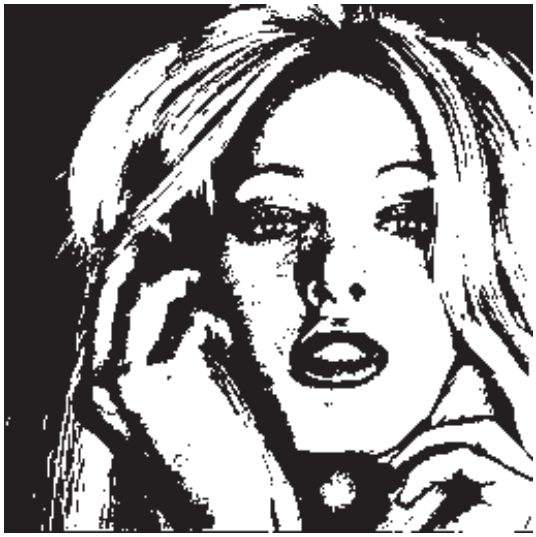
$K = \arg\min_r \{P_k(r) \geq J(m,n)\} =$

3	7	15	11
7	15	11	15
11	15	15	11
3	7	7	15

3. **20 pts**. The gray scale image $\mathbf{I}_1$ shown below has 8-bit pixels. This image was thresholded to obtain the binary image $\mathbf{I}_2$, which is also shown below. In $\mathbf{I}_2$, the pixel value 255 (WHITE) represents LOGIC_ONE and the pixel value zero (BLACK) represents LOGIC_ZERO.

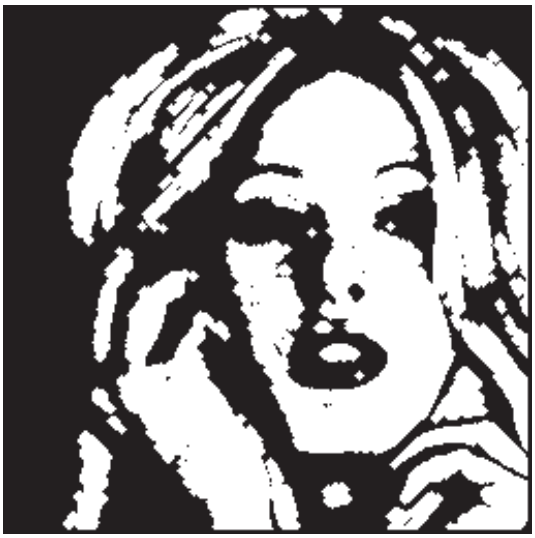
$\mathbf{I}_1$

$\mathbf{I}_2$

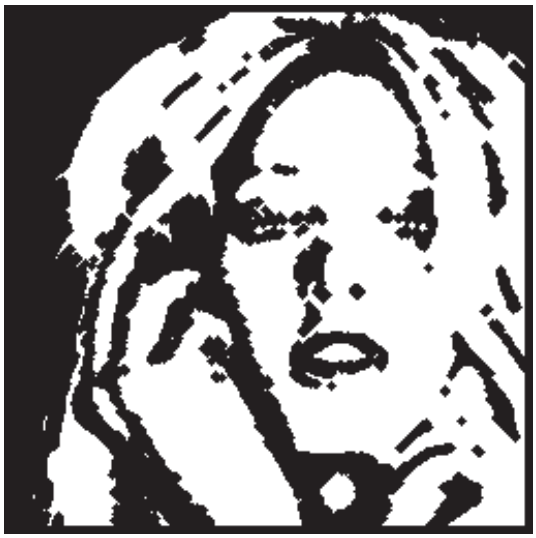


Binary morphological OPEN and CLOSE operations where performed on the image $\mathbf{I}_2$ using a 5×5 diamond-shaped structuring element. The resulting images are shown as $\mathbf{J}_1$ and $\mathbf{J}_2$ below.

$\mathbf{J}_1$ = OPEN



$\mathbf{J}_2$ = CLOSE



Determine which image is the result of the OPEN operation and which is the result of the CLOSE operation. Explain your answer in the space provided on the next page.

Workspace for Problem 3...

OPEN removes small objects and peninsulas of LOGIC ONE pixels. But it does not remove small structures composed of LOGIC ZERO pixels.

In J_1, notice that both eyebrows are preserved, but the fine white details of the hair and eyes are gone.

$$\Longrightarrow \quad J_1 = \text{OPEN}\left[I_2, B\right]$$

CLOSE removes small holes and gaps of LOGIC ZERO pixels. But it does not remove small structures composed of LOGIC ONE pixels.

In J_2, notice that fine black details of the hair are removed, but fine white details of the eyes are preserved.

$$\Longrightarrow \quad J_2 = \text{CLOSE}\left[I_2, B\right]$$

4. **20 pts.** Consider a 6×6 digital image $\mathbf{I}$ given by

$$I(m,n) = 72\delta(m,n) - 2 + \cos\left[\frac{2\pi}{6}(2m+n)\right] + 3\sin\left[\frac{2\pi}{6}(2m+2n)\right],$$

where m = column and n = row.

(a) **10 pts.** Find a closed form expression for the DFT $\widetilde{\mathbf{I}}$.

Notes p. 4.127 : $72\delta(m,n) \overset{DFT}{\longleftrightarrow} 72$

Notes p. 4.126 : $-2 \overset{DFT}{\longleftrightarrow} (-2)(36)\delta(u,v) = -72\delta(u,v)$

Notes p. 4.128 : $\cos[2\pi/6(2m+n)] \overset{DFT}{\longleftrightarrow} (\frac{1}{2})(36)[\delta(u-2,v-1) + \delta(u+2,v+1)]$

$= 18[\delta(u-2,v-1) + \delta(u+2,v+1)]$

Notes p. 4.129 : $3\sin[2\pi/6(2m+2n)] \overset{DFT}{\longleftrightarrow} j(\frac{3}{2})(36)[\delta(u+2,v+2) - \delta(u-2,v-2)]$

$= j54[\delta(u+2,v+2) - \delta(u-2,v-2)]$

$$\widetilde{I}(u,v) = 72 - 72\delta(u,v) + 18[\delta(u-2,v-1) + \delta(u+2,v+1)] + j54[\delta(u+2,v+2) - \delta(u-2,v-2)]$$

(b) **10 pts.** Show the real and imaginary parts of the **centered** DFT array in the space provided below:

$\widetilde{\mathbf{I}} =$

v \ u	-3	-2	-1	0	1	2
-3	72	72	72	72	72	72
-2	72	72	72	72	72	72
-1	72	90	72	72	72	72
0	72	72	72	0	72	72
1	72	72	72	72	72	90
2	72	72	72	72	72	72

$+ \; j \times$

v \ u	-3	-2	-1	0	1	2
-3	0	0	0	0	0	0
-2	0	54	0	0	0	0
-1	0	0	0	0	0	0
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	0	0	0	0	0	-54

5. **20 pts.** Draw lines to match the images with their log-magnitude DFT spectra.